A controlled fragment of DRT

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1   Introduction

First-order logic (FOL) is undecidable — that is, no algorithm exists that can decide whether a formula of FOL is valid or not. However, there are various fragments of FOL that are known to be *decidable*. FO^2, the two-variable fragment of FOL, is one of such languages [1, 2]. FO^2 is a first-order language where formulas have maximally two variables, no function symbols, but possibly do have equality. FO^2 has the *finite model property* [1], which means that if a formula of FO^2 is satisfiable, it is satisfiable in a finite model.

In this paper we propose a controlled fragment of Discourse Representation Theory (DRT, [3]) with a semantics formalised on the basis of the two-variable fragment with equality. DRT encapsulates the idea of text interpretation that "one and the same structure serves simultaneously as content and context" [4], where *content* refers to the semantic interpretation of sentences already processed, and *context* serves in aiding the interpretation of anaphoric expressions in subsequent sentences.

However, providing a two-variable natural language fragment is, in itself, not a new idea. In fact, the framework presented here is very much inspired by Ian Pratt-Hartmann's language E2V [5]. But as Pratt-Hartmann himself notes, E2V is "certainly not proposed as a useful controlled language". Our aim is to try to find out how useful a controlled language based on the two-variable fragment actually can be, mostly from a computational linguistic point of view. We will do this by:

- defining a transparant translation from the DRT fragment to FO^2;
- including events, thematic roles, pronouns, and plurals in the fragment;
- specifying a syntax-semantics interface.

The syntax-semantic interface of our choice will be based on combinatory categorial grammar (CCG, [6]), rather than, say, a simple definite clause grammar. Because of its type transparency principle, CCG will enable us to set up a clean interface, where each syntactic category uniformly corresponds to a semantic type. CCG gives us further means for incremental processing [6], and large databases of texts annotated with CCG-derivations are available [7], which might aid us in enlarging the lexicon for practical applications.

2 Economical DRT

The fragment of DRT that we introduce here, blessed Economical DRT, is decidable, because we can (rather straightforwardly) show that EDRT can be translated to FO^2. An EDRS (Economical Discourse Representation Structure) is an ordered pair [D,C] where D stands for the domain (a possibly empty set of discourse referents), and C is a set of EDRS-conditions. Unlike standard DRT, the domain of an EDRS has at most one discourse referent. There are only two discourse referents, named "1" and "2". This restriction doesn't mean that we can only name two different discourse referent in an EDRS — we may re-use discourse referents as often as we like, nested in sub-EDRSs. Examples below will illustrate this technique. We will use the box-notation of DRSs in examples below, or, for convenience, a flat notation in definitions and alike.

EDRS-conditions are recursively defined as follows: If P is a one-place predicate symbol, and u is a discourse referent, then P(u) is an EDRS-condition; If R is a two-place predicate symbol, and u and u' are discourse referents, then $R(u,u')$ is an EDRS-condition; If u and u' are discourse referents, then $u = u'$ is an EDRS-condition; If B is an EDRS, then +B and −B are EDRS-conditions; If B_1 and B_2 are EDRSs, then $B_1{\Rightarrow}B_2$ and $B_1{\vee}B_2$ are EDRS-conditions; Nothing else is an EDRS-condition. Most of these resemble the normal DRS-conditions, with two exceptions: −B marks negative information, and +B marks positive information.

The EDRS language is interpreted by translation to FO^2 with the aid of the function $[.]^{fo2}$. This function is defined for discourse referents, EDRS-conditions, and EDRSs. This translation always produces a formula of FO^2, simply because it only yields at most two different variables (x and y):

$$
\begin{aligned}
&[< \emptyset, \{c_1,\ldots,c_n\} >]^{fo2} = ([c_1]^{fo2} \wedge \ldots \wedge [c_n]^{fo2}) \\
&[< \{u\}, \{c_1,\ldots,c_n\} >]^{fo2} = \exists[u]^{fo2}([c_1]^{fo2} \wedge \ldots \wedge [c_n]^{fo2}) \\
&[< \emptyset, \{c_1,\ldots,c_n\} >{\Rightarrow}B]^{fo2} = ([c_1]^{fo2} \wedge \ldots \wedge [c_n]^{fo2}) \rightarrow [B]^{fo2}) \\
&[< \{u\}, \{c_1,\ldots,c_n\} >{\Rightarrow}B]^{fo2} = \forall[u]^{fol2}([c_1]^{fo2} \wedge \ldots \wedge [c_n]^{fo2}) \rightarrow [B]^{fo2}) \\
&[B_1{\vee}B_2]^{fo2} = ([B_1]^{fo2} \vee [B_2]^{fo2}) \\
&[+B]^{fo2} = [B]^{fo2} \\
&[-B]^{fo2} = \neg[B]^{fo2} \\
&[u=u']^{fo2} = [u]^{fo2} = [u']^{fo2} \\
&[R(u,u')]^{fo2} = R([u]^{fo2},[u']^{fo2}) \\
&[P(u)]^{fo2} = P([u]^{fo2}) \\
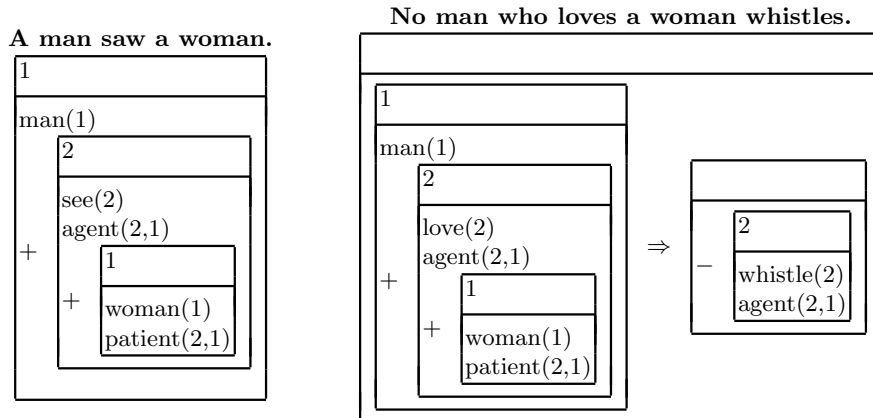&[1]^{fo2} = x \\
&[2]^{fo2} = y
\end{aligned}
$$

We borrow some terminology of standard DRT to clarify the concepts *subordination*, *accessibility*, and *insertability*. Given an EDRS B, a DRS B_1 subordinates a DRS B_2 if and only if $+B_2$ is a condition of B_1, $-B_2$ is a condition of B_1, $B{\vee}B_2$ is a condition of B_1, $B_2{\vee}B$ is a condition of B_1, $B_2{\Rightarrow}B$ is a condition of B_1, $B_1{\Rightarrow}B_2$ is a condition of B, or B_1 subordinates B, and B subordinates B_2. A discourse referent in a DRS B is *accessible* from a DRS B' if B subordinates

B′, or if B=B′. A DRS can be inserted into B (B is *insertable*) if B is not subordinated to any other DRS (i.e., is the outermost DRS), or if +B is a condition of B′ and B′ is insertable.

Accessibility is an important notion in DRT for establishing anaphoric links. Our EDRS language inherits the nice properties of accessibility of antecedents of pronouns, but also controls the use of pronouns by adding further restrictions on the structure of discourse. Insertability is the EDRT concept for conjoining sentences — below we will illustrate this mechanism.

3   Events, Thematic Roles, and Pronouns

We use a sortal ontology to distinguish between event, collection, and individual (they are all disjoint concepts). This is coded as background knowledge and can be straigthforwardly specified as axioms of FO^2. We adopt a neo-Davidsonian representation of events. Thematic roles are two-place relations between events and other entities. We use the inventory of VerbNet to assign thematic roles to verbal arguments [8], such as *agent*, *patient* and *theme*. The neo-Davidsonian representation is central to our technique of re-using discourse referents, as illustrated by the examples below.

A man saw a woman.

```
1
man(1)
    2
    see(2)
    agent(2,1)
+       1
    +   woman(1)
        patient(2,1)
```

No man who loves a woman whistles.

```
1
man(1)
    2
    love(2)
    agent(2,1)
+       1
    +   woman(1)
        patient(2,1)
```
⇒
```
        2
    −   whistle(2)
        agent(2,1)
```

There are several linguistic constraints implied by this fragment. First of all, a sentence can have at most one universal quantifier (triggered by, for instance, *every* or *no*). The current syntax-semantic interface requires this to be the subject noun phrase. Similarly, a sentence can have only one pronoun referring back to an antecedent in a previous sentence. Different insertability possibilities allow a pronoun to have several possible antecedents. For instance, in the example above, a DRS for the sentence "She smiled" could be inserted in the deepest embedded +DRS, thereby establishing an anaphoric link between the third-person pronoun and "a woman". Proper names (and definite descriptions) trigger a uniqueness presupposition, accommodated as part of the background knowledge. Uniqueness statements can obviously be represented with two variables — an an example consider the proper name *Lou* and the presupposition $\forall x \forall y ((lou(x) \wedge lou(y)) \rightarrow x=y)$.

4   The Syntax-Semantics Interface

To obtain syntactic structure we employ a categorial grammar with basic categories n, np, s, pp (the notation of slashes follows CCG). Each syntactic category corresponds to a (typed) partial EDRS. Because we only have two different discourse referents, β-conversion with renaming of variables to overcome accidental capture of free variables is not needed — instead it is safe to use unification for substitution. We will do so with a two-place operator (A·B), similar to lambda-abstraction, where B is always an EDRS, and A discourse referent or another dot operation. Partial EDRSs can also only contain at most two distinct discourse referents.
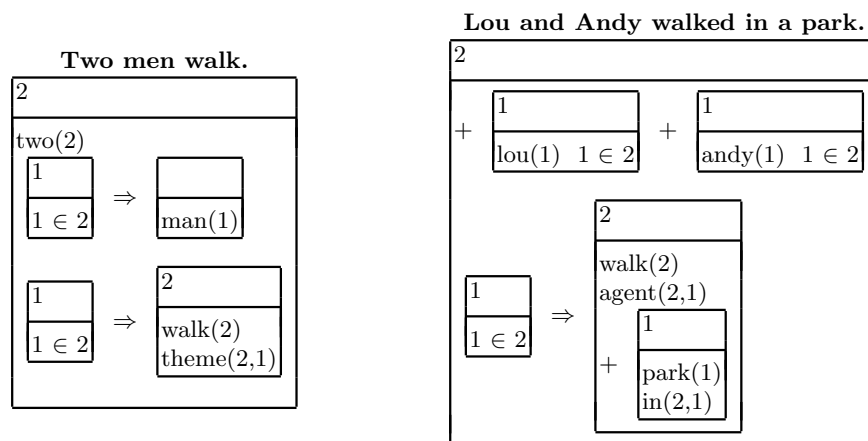
Cat	Partial EDRS	Tokens
n	$(1 \cdot \langle \emptyset, \{\text{car}(1)\} \rangle)$	car
n/n	$(1 \cdot B) \cdot (1 \cdot \langle \emptyset, \{\text{big}(1), +B\} \rangle)$	big
np	$(1 \cdot B) \cdot \langle \{1\}, \{\text{person}(1), +B\} \rangle$	someone
s/(s\np)	$((1 \cdot B) \cdot \langle \emptyset, \{\text{female}(1), +B\} \rangle) \cdot C) \cdot C$	she
np/n	$(1 \cdot C) \cdot (1 \cdot B) \cdot \langle \{1\}, \{+C, +B\} \rangle$	a
(s/(s\np))/n	$(1 \cdot C) \cdot (((1 \cdot B) \cdot \langle \emptyset, \{\langle \{1\}, \{+C\} \rangle \Rightarrow B\} \rangle) \cdot D) \cdot D)$	every
s\np	$((1 \cdot \langle \{2\}, \{\text{walk}(2), \text{theme}(2,1)\} \rangle) \cdot C) \cdot C$	walks
(s\np)/np	$((1 \cdot \langle \emptyset, \{\text{th}(2,1)\} \rangle) \cdot B) \cdot (((1 \cdot \langle \{2\}, \{\text{see}(2), \text{ag}(2,1), +B\} \rangle) \cdot C) \cdot C)$	saw
pp	$(2 \cdot \langle \{1\}, \{\text{london}(1), \text{to}(2,1)\} \rangle)$	to London

In this version of EDRT we only use three combinatory rules: forward application (FA), backward application (BA), and forward composition (FC). FA is defined as follows: α/β:(X·Y) β:X yields α:Y. BA is defined analogously. FC is defined as: α/β:(X·Y) β/γ:Z·X yields α/γ:Z·Y. Forward type-raising is also part of the machinery.

5   Adding the Plural

We adopt a theory of collections to model plurals [9]. Membership is stated by a two-place relation between individuals and collections, for convenience designated by the $\in$ symbol. Two example EDRSs with respectively a counting quantifier and summation, are shown on the next page. Both examples illustrate the distributive reading. The collective reading can also be obtained by relating the thematic role of the event directly with the discourse referent denoting the collection. The interpretation of the counting quantifier is defined by meaning postulates and part of the background knowledge. Here we show how it can be done by using at most two variables for the cardinal *two*. The first of these axioms says that a collection with the property "two" has two members, and the second states that these are distinct members. It should be clear that the method can be extended to numerals with higher cardinality, without resorting to more than two variables.

$\forall x(\text{two}(x) \rightarrow (\exists y \ \text{first-member}(x,y) \wedge \exists y \ \text{second-member}(x,y)))$
$\forall x(\text{two}(x) \rightarrow \neg \exists y(\text{first-member}(x,y) \wedge \text{second-member}(x,y)))$
$\forall x \forall y(\text{first-member}(x,y) \rightarrow x \in y) \qquad \forall x \forall y(\text{second-member}(x,y) \rightarrow x \in y)$

Two men walk.

Lou and Andy walked in a park.

6 Final Remarks

The presented DRT fragment is of course far more restricted than the full blown version of DRT (EDRT cannot represent donkey sentences). Nevertheless, we hope to have shown that the two-variable fragment has some potential for (quasi) natural language applications. For instance, the use of neo-Davidsonian event-style semantics has no restrictions on the number of modifiers (but it does on pronouns or universal quantified noun phrases).

This is work in progress. Space limitations forced us to leave out a number of issues. In a more elaborated paper we plan to further illustrate incremental parsing, the generation of background knowledge axioms, and the syntax-semantic interface for plurals.

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