A Semantics for Nominal Comparatives

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Abstract
This work adopts the perspective of plural logic and measurement theory in order first to focus on the microstructure of comparative determiners; and second, to derive the properties of comparative determiners as these are studied in Generalized Quantifier Theory, locus of the most sophisticated semantic analysis of natural language determiners. The work here appears to be the first to examine comparatives within plural logic, a step which appears necessary, but which also harbors specific analytical problems examined here.

Since nominal comparatives involve plural and mass reference, we begin with a domain of discourse upon which a lattice structure (Link's) is imposed, and which maps (via abstract dimensions such as *weight in kilograms*) to concrete measures (in N,R+). The mapping must be homomorphic and Archimedean. Comparisons begin as simple predicates on dimensions or measures; from these we derive classes of predicates on the domain, i.e., generalized determiners (quantifiers), and show, e.g., how monotonicity properties follow in the derivation. This results in a proposal for a logical language which includes DERIVED determiners, and which is an attractive target for semantics interpretation; it also turns out that some interesting comparative determiners are first order, at least when restricted to nonparametric and noncollective predications.

1 Introduction

Nominal comparatives are syntactically and semantically complex, involving complexes of constraints, (under)specification, and quantification. Their semantics is further complicated by the fact that they necessarily involve plural and mass reference. The list below is representative of the syntactic and semantic range of nominal comparatives.

More (fewer) than 7 children sang.
How many children sang?
A trained 7 more (fewer) children than B saw (dogs).
A trained twice as many children as B saw (dogs).
A trained at least twice as many children as B saw (dogs).

More (less) than 2 liters of water spilled.
How much water spilled?
How many liters of water spilled?
A spilled two liters more (less) beer than B drank (water).
A spilled twice as much beer as B drank (water).
A spilled at least twice as much beer as B drank (water).

The present section provides an overview of the paper and a review of previous work. The following section includes all of the basic logical apparatus, including the relevant assumptions about plural and mass ontology, the requirements on measure theory, and the basic function of determiners derived from measure specifications (including useful subcases). Section 3 reviews and derives the properties of measure determiners studied in generalized quantifier theory (hence GQT), and Section 4 sketches a logical language built on these ideas. Section 5 explores extensions to parametric determiners, determiners derived from additive relations, and determiners derived from multiplicative relations. Sections 6 reports on a computational implementation.

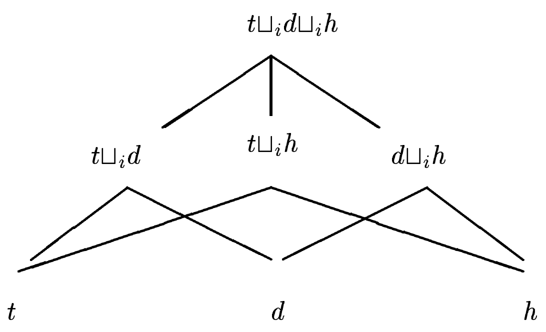

Figure 1: Semilattice of plural entities (for 3 individuals). '$\sqcup_i$' is lattice join; and 'a $\sqsubseteq_i$b' or 'a is *part-of* b' holds if one can travel from a up to b along $\sqcup_i$-lines. E.g. $d \sqsubseteq_i t \sqcup_i d \sqcup_i h$.

1.1 Previous Work

Although there's an extensive literature on the semantics of ADJECTIVAL comparison, there's much less on NOMINAL comparatives. Keenan and Moss 1984 investigate these fairly abstractly, also from a GQ perspective, demonstrating e.g. conservativity (and adducing an interesting class of ternary determiners). But their approach is broad and systematic; comparative determiners are syncategorematic. The treatment below is more detailed.

Cartwright 1975, ter Meulen 1980 and others have pursued measure-theoretic analyses of mass-terms and plurals, but without assuming a lattice-structured ontology. Link 1987 discusses quantification over plural domains in a way largely compatible with the present proposal, which, however, generalizes his definitions. The present approach is closest to Krifka 1989, but this appears to be the first application to comparison and its relation to quantification.

2 Measures and Determiners

Although the current proposal is intended to extend to mass measurement, we focus on plurals throughout the presentation.[1]

2.1 Representation of Plurals

Since nominal comparatives crucially involve plural and mass term reference, we need to proceed from a representation language which accommodates it. This is a crucial respect in which the present study differs from other work on the semantics of comparatives, e.g. Klein 1981, von Stechow 1984, Keenan and Moss 1984, Ballard 1989, or Rayner and Banks 1990, and, as will be demonstrated in Section 2.3 below, the incorporation of plural logic complicates the analysis of comparatives due to its postulation of monotonicity properties (distributivity) inherent in certain plural predications.

In presenting a plural logic we borrow from the now extensive literature on the logic of plurals and mass terms (Cf. Link 1983, Link 1987; Lønning 1987, Lønning 1989b, Lønning 1989a). The structured plural ontology is independently motivated and requires no special modifications for the analysis of comparatives.

1. Space prohibits examining mass reference separately. However, the generalization to mass reference is standard and straightforward in lattice-based theories—mass term lattices are not atomic, while plural lattices are.

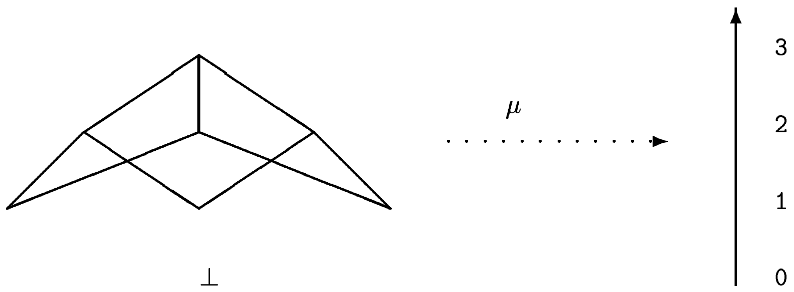

Figure 2: Measure functions μ maps elements of the domain, E, onto ordered measures (of various dimensions)

We begin therefore with a universe of discourse E upon which Link's familiar semilattice structure has been imposed, ordered by '$\sqsubseteq_i$', or '*part-of*'. This is the relation that holds both between subgroups and groups, and also between individuals and groups containing them. The atoms in the lattice satisfy the predicate $\text{atom}(x)$, and they correspond to individuals; the nonatomic elements correspond to groups of individuals. Cf. Figure 1. We use D to designate the set of the atoms in E—corresponding to the nongroup individuals; this is appropriate since D corresponds to the normal domain of discourse. We'll let $x\,\text{atom-}\sqsubseteq_i y \;\Leftrightarrow\; \text{atom}(x) \wedge x \sqsubseteq_i y$. Link furthermore requires that the lattice be complete, so that we may allow, for any predicate of atoms P, that $*P$ denote the predicate true of the supremum of $[\![P]\!]$ (the least upper bound under $\sqsubseteq_i$of the atoms in $[\![P]\!]$) and everything beneath it, i.e. $[\![*P]\!] = \{x|x\sqsubseteq_i \vee [\![P]\!]\}$; We shall also have occasion to use Link's distributive predicate operator, ^{D}; for any predicate P, $^DP(x) \leftrightarrow \forall x'(x'\text{atom-}\sqsubseteq_i x \rightarrow P(x'))$, i.e. ^DP is true of objects whenever P is true of their component atoms.

We assume that common noun (CN) phrases denote predicates of the $*P$ sort, and that NPs, interpreted as GQs, obtain their restrictors from the interpretation of their $\bar{\mathbf{N}}$ heads. We thus ignore the few CN phrases (e.g., *pair, dozen*) which are restricted to nonatomics of a particular size.[2] It is of course fine for there to be (determined) NPs which are interpreted not as GQs, but rather as simple referring expressions, as in Discourse Representation Theory. Cf. Nerbonne et al. 1990 for a development of plural logic with indefinite referring expressions. But this complication will not concern us below.

2.2 Plurals, Mass Terms and Measures

Plural and mass objects are MEASURABLE; for plural objects, cardinality is the salient measure, for mass objects, weight and volume are normally the more useful measures. Figure 2 illustrates the function of measuring: mapping a structured domain onto a set of MEASURES: $\mu : E \mapsto \mathcal{M}$, where $\mathcal{M}$ is an ordered set of measures, ordered by $\leq_{\mathcal{M}}$.

It should be clear from the examples and from Figure 3 that we are only concerned here with measures of size—number of members, weight or volume. All of these respect homomorphically the *part-of*-relation on the structured domain. These seem to be the only measures which function REGULARLY as natural language determiners—in contrast to the many other measures which we find in adjectival comparison and specification, e.g., *five degrees warmer* (temperature),

2. N.b. in the present treatment quantified logical forms have variables which range over plural entities, but they have truth conditions which depend on the lattice atoms, and atomic quantities. Link 1987 discusses the need for genuinely quantifying over the plural entities.

two meters tall (height) or *several points better* (test results). This seems worthy of separate remark:

Homomorphic measure determiners Only homomorphic measures serve to define natural language determiners, i.e. where μ is a homomorphism from $< E, \sqsubseteq_i >$ $(< [\![\bar{\mathbf{N}}]\!], \sqsubseteq_i >)$ to $< \mathcal{M}, \leq_{\mathcal{M}} >$

The basic insight is due to Krifka 1989, who, on the basis of the contrast between *twenty grams/*carats of gold*, postulated that so-called numerative constructions need additive measure functions.

Of course we do find nonsize measures used SPORADICALLY in deriving determiners, but interestingly, these uses seem restricted to cases in which the measure-ordering does preserve the *part-of*-relation, at least over the substructure defined by the head noun: *five dollars worth of gas* (cost) , *one megawatt of electricity* (energy). The homomorphism requirement is more properly restricted to the $\bar{\mathbf{N}}$ denotation. Alternatively, one might examine so-called SIMILARITY TRANSFORMATIONS (cf. Krantz et al. 1971, p.10) to proper scales.

In natural language determiners measures may be specified as simple numbers (cardinalities), but they are more generally specified as pairs: 2.2 *lb.*, 1 *kg.*, etc. We understand the latter element as specifying the mapping, the former the value under that mapping. It is clear that measure mappings should respect the plural/mass structure, e.g. that the measure of the sum of (nonoverlapping) objects should be the sum of the component measures. The relevant requirements are the following:

μ is a *measure* function $\stackrel{def}{=}$

$\mu : E \mapsto \mathcal{M}$ e.g. $\mathcal{M} = \mathrm{N}, R^+ \cup 0$

$x \sqsubseteq_i y \wedge \mu(x) \neq 0 \rightarrow \exists n > 0\ n \cdot \mu(x) \geq_{\mathcal{M}} \mu(y)$ **Archimedean**

$x \sqcap_i y = \bot \rightarrow \mu(x \sqcup_i y) = \mu(x) + \mu(y)$ **Additive**

The first clauses repeats the homomorphic requirement, discussed above. Krantz et al. 1971 argue that the second is assumed in all normal measurement; it frees measurement from ties to any particular scale. We make no explicit appeal to it below. We shall have recourse to the third clause below in analyzing both additive measures *Two more (grams of) X than CONDITION* and multiplicative measures (*Twice as many/much X as CONDITION*). The additive axiom allows that we add repeatedly, allowing thus multiplicative conditions, as well:

$$\underbrace{\mu(x) + \cdots + \mu(x)}_{n \text{ times}}$$

These requirements are standardly imposed on EXTENSIVE measures, in measurement theory, Krantz et al. 1971, 71ff. Manfred Krifka has advocated their application in event semantics in several important papers: Krifka 1987, Krifka 1989, Krifka 1990. Krifka's work is especially *à propos* because it likewise postulates a structured ontology as the domain of the measurement function.

It is worth pointing out that measure functions, as the definition here has it, perform two subtasks which might profitably be distinguished:[3] (i) measure functions divide the domain of discourse into equivalence classes of entities having the same measure; and (ii) they assign concrete measures to (the members

3. Cf. Cresswell 1976, pp.280-85 for a similar point.

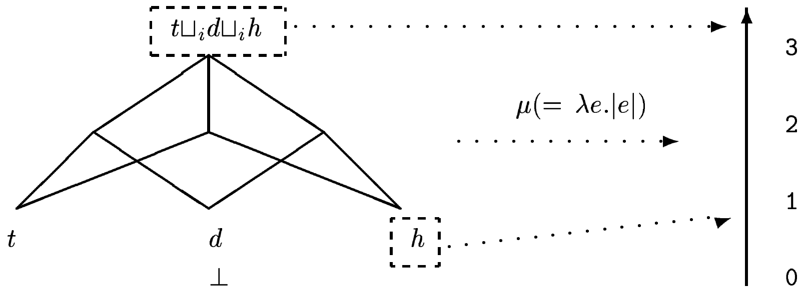

Figure 3: Cardinality is a measure function over the plural lattice.

of) those equivalence classes, e.g. 2.2 *lb.* The advantage in using this division of labor would be the opportunity to infer that two objects had the same weight even given the premises e.g. that one weighed 2.2 *lb.* and the other 1 *kg.* We could then make formal sense of the same abstract weight having more than one measure. In the interests of presentational simplicity I eschew this division of labor here, however.

Cardinality is a measure function in the sense defined above. It maps the plural domain to N, and is additive in the required sense. The requirement that measures be additive (and multiplicative) is needed in particular to treat some of the complex measure phrases we examine in Section 5.2.

It seems clear that measure phrases can be used to denote measures:

Three pounds is more than a kilogram.

And I take this possibility as justifying somewhat the postulation of the measure sets and functions into them. But our focus lies on the on the descriptive semantics of these measures phrases in their use as determiners in noun phrases, not on philosophical and foundational questions about the ontological status of measures—whether primitive or derived,[4] I assume the existence of measures as well as their participation in a rich mathematical structure in order to explicate the semantics of their use as determiners.

2.3 Deriving Determiners from Measures—Problems

In general, measures contribute to quantifiers by providing RESTRICTORS. The basic idea is simple: given a measure m we wish to derive a determiner D_m. This is accomplished by obtaining the inverse image of m under a measure function μ, i.e. $\mu^{-}(m)$, which is then in turn available to restrict an determiner. We examine difficulties with a natural formulation based on the existential; similar problems arise using the universal:

$$DET_m\, x\,(R(x)\,,\, S(x)) \quad \text{iff} \quad \exists\, x\,(x \in \mu^{-}(m)\, \wedge\, R(x)\,,\, S(x))$$

We are assuming the view of quantifiers from GQT (Westerståhl 1989)—that of relations between properties, represented above as '$R(x)$' and '$S(x)$', which are mnemonic for RESTRICTOR and SCOPE. [5] We retain the relational notation '$Qx(R(x), S(x))$' because it is common in GQT. Since we defend the thesis

4. But cf. Cresswell *op.cit.* on how some primitive properties of measures may be derived.
5. Note that, where Westerståhl writes $DET(A, B)$, we write $DETx(A(x), B(x))$. The latter notation alleviates some of the need for λ's in representing natural language meanings. Nothing more is intended by the modification.

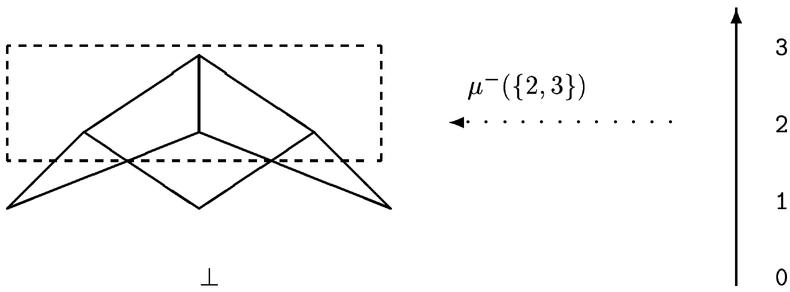

Figure 4: $\mu^{-}(\{2,3\})$, image of $\{2,3\}$ under the inverse measure function.

below that measure quantifiers can be reduced to properties of properties, we could write '$Qx(R(x) \wedge S(x))$', but we stick with the standard notation.

This simple view given by the above proposal is complicated (i) by comparatives and other modifiers of measurement phrases, which involve not just single measures, but various SETS of measures; and (ii) by the plural structure on the domain of discourse E, in particular the condition on (distributive) predicates that they be closed under $\sqcup_i$. We take up these issues in turn.

Comparatives (*more than one* or *more than one ounce*) refer not to a single measure, but to specified sets of measures (those greater than one or those greater than one ounce). Allowing reference to sets of measures is straightforward, however. Figure 4 illustrates the obvious generalization from taking the inverse of a single measure to taking the image of a set of measures under the inverse measure function. The definitions below map SETS of measures onto determiners. For example, we could now generalize the definition above: For $M \subset \mathcal{M}$ define binary DET_M:

$$DET_M \overset{def}{=} \lambda R, S.\exists x(x \in \mu^{-}(M) \wedge R(x),\ S(x))$$

or equivalently

$$\lambda R, S.\exists x(R(x) \wedge \mu(x) \in M,\ S(x))$$

This construal, too, needs to be generalized to deal with the second problem, i.e. the interaction of plural structure and comparative determiners, noted in Link 1987. This may be seen in examples such as the following:

Fewer than three children sang. (1)

Let's assume that $[\![sang]\!]$ (like other simple predicates) is closed under $\sqcup_i$, so that if $x, y \in [\![sang]\!]$ then $x \sqcup_i y \in [\![sang]\!]$. This means e.g. that if the 2-sets in Figure 4 are in $[\![sang]\!]$, so is their join, the 3-set. But, by the simple derivation of measure determiners proposed above, sentence (1) could be true, since there's a 2-set with the required properties! This is is clearly incorrect, and, moreover, it's the direct result of working in the plural structure. (This is the problem referred to in Section 2.1 above, and it is simply ignored in treatments which abstract away from the problems of plural and mass term reference.) It should be clear that the natural alternative to the use of the existential in definition above, the universal, has symmetric difficulties with measure phrases such as *more than three*.

The plural structure must inform the derivation of determiners from measures and measure sets. In this case, we'd like the result that *fewer than n* holds of P, Q just in case there's no entity of size n or greater such that P and Q may be predicated of it.

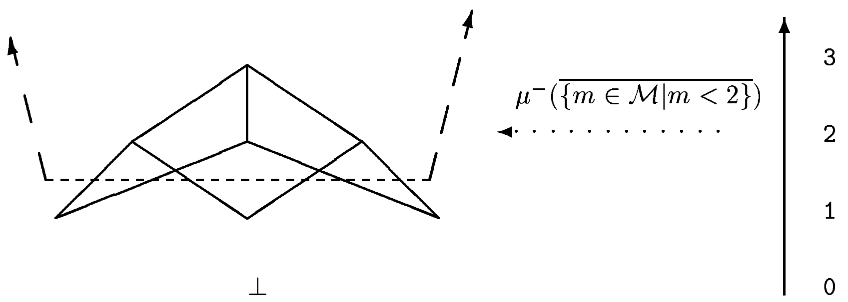

Figure 5: Inverse image of complement of $\{m \in \mathcal{M}|m < 2\}$, downwardly closed measure set. Derived determiner "fewer than 2" lives on this image.

2.4 Determiners Derived from Measure Sets

The general scheme for deriving determiner meanings from the specification of measure sets is as follows:

$$\begin{array}{l} \text{Let } \mathcal{M} \text{ be the range of a measure function, ordered by } \leq_{\mathcal{M}}, \text{ and} \\ \text{let } M \subseteq \mathcal{M}. \text{ We obtain the determiner derived from } M,\ DET_M : \\ DET_M\, x\,(R(x), S(x)) \stackrel{def}{=} \mathbf{max}_{\leq_{\mathcal{M}}}\{\mu(x)|R(x) \wedge S(x)\} \in M \end{array} \tag{2}$$

I.e. among the measures in M is the maximal measure of objects satisfying the predicates R and S. Note that this handles the "fewer than 3" as well as the "more than 3" cases. The "fewer than 3" case comes out right because the definition here requires that the maximal measure (max) satisfying the properties involved falls within the measure set.[6]

The more exact properties of determiners depend on the properties of the sets of measures M they're based on, especially whether M is be upwardly (downwardly) closed, or convex.

$$\begin{array}{l} M \text{ is } \uparrow\text{-closed iff } m \in M \wedge m \leq_{\mathcal{M}} m' \rightarrow m' \in M \\ M \text{ is } \downarrow\text{-closed iff } m \in M \wedge m' \leq_{\mathcal{M}} m \rightarrow m' \in M \\ M \text{ is convex iff } m, m'' \in M \wedge m \leq_{\mathcal{M}} m' \leq_{\mathcal{M}} m'' \rightarrow m' \in M \end{array} \tag{3}$$

The three interesting subcases, where the initial set of measures M is $\uparrow$-closed ("more than 3", "more than 5 kg"), $\downarrow$-closed ("fewer than 3","less than 3 kg"), or convex ("between 3 and 7") allow interesting reductions.

M **$\uparrow$-closed** Then:

$$\mathbf{max}_{\leq_{\mathcal{M}}}\{\mu(x)|R(x) \wedge S(x)\} \in M \leftrightarrow \exists x (R(x) \wedge \mu(x) \in M, S(x))$$

$\rightarrow$: The max must measure some $x \in E$.
$\leftarrow$: Trivial consequence of $\uparrow$-closure.
Thus the definition initially given holds for this subclass.

M **$\downarrow$-closed** Then:

$$\mathbf{max}_{\leq_{\mathcal{M}}}\{\mu(x)|R(x) \wedge S(x)\} \in M \leftrightarrow \neg\exists x (R(x) \wedge \mu(x) \in \overline{M}, S(x))$$

$\rightarrow$: Since M is $\downarrow$-closed, $M <_{\mathcal{M}} \overline{M}$, and in particular $\mathbf{max}_{\mathcal{M}} < \overline{M}$.

6. For measure sets in the reals, maxima may not exist, so that we might prefer to use suprema rather than maxima. On the other hand, we'll never measure such suprema, so that this may be a nicety. The proof of reduction below (this section) for the case of $\uparrow$-closed M requires that maxima, not suprema be available.

There can't be any y satisfying R, S with $\mu(y) > \mathbf{max}_{\mathcal{M}}$, so none $\in \overline{M}$
$\leftarrow$: If there's no x with $R(x) \wedge S(x) \wedge \mu(x) \in \overline{M}$, then every x with $R(x) \wedge S(x)$ is such that $\mu(x) \in M$, including $\mathbf{max}_{\mathcal{M}}$

M **convex** (and neither $\uparrow, \downarrow$-closed) Then:
$\mathbf{max}_{\leq_{\mathcal{M}}} \{\mu(x)|R(x) \wedge S(x)\} \in M \leftrightarrow$
$\exists\, x(R(x) \wedge \mu(x) \in M, S(x)) \ \wedge\ \neg\exists\, x(R(x) \wedge \mu(x) \in \overline{\{m|m < \wedge M\}}, S(x))$

$\rightarrow$: Given that the **max** is in M, then (i) something is, and (ii) nothing measures past M, and therefore past its supremum. (Convexity is irrelevant in this direction.)
$\leftarrow$: We can't simply reverse this reasoning, however. We might have a set of measures M into which some x satisfying R, S measures, and such that no satisfactory x measures beyond it. But if the measure set isn't convex, we may run afoul of plural structure again, i.e. we may have found some nonmaximal x satisfying R, S which measures into M (consider *an even number of*). On the other hand, if M is convex, then the **max** must be found within it.

We don't propose further reductions, and we shall not discuss the remaining class, that of nonconvex measure sets in any detail.

The significance of the reductions is twofold. On the one hand, they will be useful when it comes to adducing monotonicity properties, because they amount to reductions to the well-studied existential and negative existential quantifiers. More interestingly, from the point of view of design for meaning representation languages, the reductions show that the properties of complex determiners ("more than 3") arise from the (closure) properties of the measures sets they are derived from. Since these in turn are inherent in comparison, we have an opportunity to derive complex determiner meanings (and their inferential properties) from the type of comparison involved.

2.5 Simple Examples

We showed above that determiner definitions follow once measure sets are provided. This is quite general; the definitions are available not only for measure sets provided by comparative phrases, but for measure sets quite generally. It is now time to return to the treatment of comparatives, in order to illustrate how the closure properties of measure sets (investigated in the last section) can be put to use. In each case, we assume information about closure properties in order to provide reasonable determiner definitions.

- For $\uparrow$-closed M, define binary $\uparrow DET_M$:

$$\uparrow DET_M \stackrel{def}{=} \lambda R, S.\exists x(R(x) \wedge \mu(x) \in (M), S(x))$$

Natural language examples include: *More than 2 children sang, More than 2 liters of water spilled, At least 2 children sang,* etc.

- For $\downarrow$-closed (and not $\uparrow$-closed) M, let $\overline{M}$ be the complement wrt $\mathcal{M}$, and define binary $\downarrow DET_M$:

$$\downarrow DET_M \stackrel{def}{=} \lambda R, S.\neg\exists x(R(x) \wedge \in \mu(x) \in \overline{M}, S(x))$$

Natural language examples include: *Fewer than 2 children sang, Less than 2 liters of water spilled, At most 2 children sang, Not more than seven children sang,* etc.[7]

7. But Carl Pollard has brought one sticky case to my attention:

- For M convex, (but neither $\downarrow$-closed nor $\uparrow$-closed):

$$\not\uparrow\not\downarrow DET_M \stackrel{def}{=} \lambda R, S. \uparrow DET_M x(R(x), S(x)) \\ \wedge \ \downarrow DET_{\{m \in \mathcal{M} | m \leq \wedge(M)\}} x(R(x), S(x))$$

Cf. *Exactly five children sang, Between ten and twenty children sang, Either four or five ..., At least three and not more than seven ...,*

3 Properties of Measure Determiners

The section reviews the properties of the derived quantifiers against the background of GQT. van Benthem 1983, p.451 singles out CONSERVATIVITY as a general property of natural language determiners:

$$\textbf{CONS} \qquad DETx(A(x), B(x)) \Leftrightarrow DETx(A(x), A(x) \wedge B(x))$$

It thus comes as no surprise that the general definition of measure determiners (2) guarantees that they are conservative. This is a straightforward consequence of the definition (repeated here):

$$DET_M\, x\,(R(x), S(x)) \stackrel{def}{=} \mathbf{max}_{\leq_{\mathcal{M}}} \{\mu(x) | R(x) \wedge S(x)\} \in M$$

which is conservative, since:

$$\mathbf{max}_{\leq_{\mathcal{M}}} \{\mu(x) | R(x) \wedge S(x)\} = \mathbf{max}_{\leq_{\mathcal{M}}} \{\mu(x) | R(x) \wedge (S(x) \wedge R(x))\}$$

It is similarly straightforward to see that all the measure determiners respect QUANTITY (van Benthem 1983,456):

QUANT $\quad DET_E x(A(x), B(x))$ only depends on the number of individuals in $A, A \cap B, B,$ and E[domain]

Though of course here we should prefer to generalize "number" to "measure". Finally, measure determiners must be symmetric:

$$\textbf{SYMM} \qquad DETx(A(x), B(x)) \Leftrightarrow DETx(B(x), A(x))$$

We can likewise adduce the monotonicity of the comparative determiners (those derived from $\uparrow$ / $\downarrow$-closed measure sets):

$$\textbf{MON}\uparrow \quad DETx(A(x), B(x)) \wedge \forall x(B(x), B'(x)) \Rightarrow DETx(A(x), B'(x))$$
$$\textbf{MON}\downarrow \quad DETx(A(x), B(x)) \wedge \forall x(B'(x), B(x)) \Rightarrow DETx(A(x), B'(x))$$

The **MON**$\uparrow$ of $\uparrow DET_M$ follows from the **MON**$\uparrow$ of the existential, which it is was shown to reduce to (Section 2.5). Similarly, $\downarrow DET_M$ inherits **MON**$\downarrow$ from the **MON**$\downarrow$ of the negative existential, its reduction. If we investigate monotonicity in the first argument of the determiner, we see that $\uparrow DET_M$, like the existential, is $\uparrow$**MON**, or $\uparrow$PERSISTENT, while $\downarrow DET_M$, like the negative existential, is $\downarrow$**MON**, or $\downarrow$PERSISTENT (van Benthem 1983, 452-3 has the relevant definitions and references.) Keenan and Moss 1984, 86ff make similar remarks about comparatives determiners, but we have we have shown how they follow from a unified view of measure determiners as predicates on measures.

Consider a situation where there are multiple meetings of kids. Interlocutor A wishes to emphasize that these were big meetings, while B wants to provide a counterexample:

A: No fewer than 5 kids met in every meeting.

B: Wrong. Only 4 kids met on the playground. SO FEWER THAN 5 KIDS DID MEET.

B is clearly NOT denying that there were kids' meetings of size 5 or more, but claiming that at least one was smaller. This is the existential reading.

I think the example CAN have the apparently counterexemplary reading, but that it should be possible to analyse the example in keeping with the proposal here if the (event) quantification over meetings is seen to be infecting the interpretation, i.e. if it's analyzed roughly "There was a meeting ...".

3.1 An Algebraic Perspective

Finally, an algebraic perspective is interesting. The quantifiers generated by determiners (once a restrictor property has been provided) exist within a lattice of properties, and these are likewise studied. We interpret the predicates via their set-theoretic extensions and examine the structure generated by the subset relation. For example, the negative existential not only satisfies **MON**$\downarrow$, as noted above, it is also closed under $\sqcup$ (disjunction), i.e., for given A

$$\neg\exists x(A(x), B(x)) \;\wedge\; \neg\exists x(A(x), B'(x)) \Rightarrow \neg\exists x(A(x), B(x) \vee B'(x))$$

So if *no one swims* and *no one jogs*, then it follows that *no one swims or jogs*. These two characteristics, downward closure and closure under $\sqcup$, show the (scope of the) negative existential to be an algebraic IDEAL (Burris and H.P.Sankappanavar 1982, 155). Since the the $\downarrow DET_M$ is reducible to the negative existential, it too is an ideal, given the analysis. But this result may appear questionable given the following fallacy:

Fewer than 4 people swim.
Fewer than 4 people jog.
Fewer than 4 people swim or jog.

The definitions thus far would appear to make the fallacy valid, as may be appreciated formally by inspecting the logical rendering of the conclusion:

$$\neg\exists\, x(\text{person}(x) \wedge \mu(x) \geq 4, \text{swim}(x) \vee \text{jog}(x))$$

There can't be any x of the sort required to refute this, unless x simultaneously satisfies the measure restriction AND one of the properties in the disjunctive scope as well. But the same x would refute one of the premises.

The key to understanding the argument is the plural structure; there may be no x s.t. $P(x)$ and $\mu(x) > m$, and no y s.t. $P'(y)$ and $\mu(y) > m$, even while there is a z, *viz.* $x \sqcup_i y$ that is of the larger size. And while this plural object can't satisfy a disjunction without satisfying one of its disjuncts, it CAN satisfy a closely related property, e.g.

$$\lambda z \forall z'(z' \sqsubseteq_i z \;\rightarrow\; \text{swim}(z') \vee \text{jog}(z'))$$

equivalently:

$$^{D}\lambda z(\text{swim}(z) \vee \text{jog}(z))$$

If this is the meaning of the disjunctive VP, then we can explain the fallacy, even while maintaining the analysis thus far given. This may seem like a radical proposal, but we needn't necessarily revise the semantics of disjunction to do this, since it would suffice to identify any source for the distributive reading, e.g. a generally available option in VP interpretation, as Roberts 1987, suggests. See Link 1983 for a related proposal on the semantics on predicate CONJUNCTION for examples such as *husband and wife*.[8]

The $\uparrow DET_M$ cases are clearly **MON**$\uparrow$, as noted above, and they just as clearly fail to be closed under $\sqcap$, so that they constitute QUASI-FILTERS, rather than FILTERS, just as singular existential determiners are. The following table summarizes the quantifier properties adduced in this section.

8. It's also worth noting here that we also have a further option to explore, *viz.* using the "distributive" join directly in the model theory (rather than viewing it as a property of the mapping from natural language into the model. I have not explored this in depth, however. At this point I would prefer the solution proposed by Roberts.

Q	**CONS**	**QUANT**	**MON**	closure	algebraic character
$\exists_{sg}$	+	+	$\uparrow$ **MON** $\uparrow$	$-(\sqcap)$	quasi-filter
$\uparrow DET_M$	+	+	$\uparrow$ **MON** $\uparrow$	$-(\sqcap)$	quasi-filter
$\neg\exists_{sg}$	+	+	$\downarrow$ **MON** $\downarrow$	$\sqcup$	ideal
$\downarrow DET_M$	+	+	$\downarrow$ **MON** $\downarrow$	$\sqcup$	ideal

4 A Logic with Measure Determiners LMD

Section 2.4 shows that measure determiners are well-defined semantically. In this section we suggest a form for measure determiners in a meaning representation language.

In order to specify a set of measures, all that is really required is a relation, e.g. *more than*, and a pole of comparison—a fixing of one term in the relation, which is effectively interpreted as a supremum in the measure set, e.g. '3 *l*' (*three liters*). We can designate this, e.g., via '$(> 3l)$'. Section 2.4 demonstrates that, given a set of measures, we can properly derive a determiner meaning. It should be enough, therefore, to represent *More than 3 liters of water spilled* as

$$(> 3l)\, x\, (\text{water}(x), \text{spilled}(x))$$

This compact form, in which we'd like to represent the meaning of measure determiners, is definable given the notion of derived determiner above.

We assume reference to numbers (and magnitudes) and reference to measures via common name, e.g. 3, 4 *kg.*, etc. MEASURE SPECIFIERS denote the relations on which set descriptions depend, and MEASURE DETERMINERS are formed by combining specifiers and measures:

$$\begin{array}{rcl} <\text{magnitude}> & ::= & <\text{digit}>*.<\text{digit}>* \\ <\text{scale}> & ::= & \text{kg, l, g, lb, \ldots} \\ <\text{measure}> & ::= & (<\text{digit}>*\,|\,<\text{magnitude}>)\;\;<\text{scale}> \\ <\text{measure-specifier}> & ::= & >, \geq, <, \leq, =, \neq \\ <\text{measure-determiner}> & ::= & \texttt{"("}<\text{measure-specifier}><\text{measure}>\texttt{")"} \end{array}$$

Westerståhl 1989 provides basic definitions for a language of generalized quantifiers, to which we propose the above extension. Measure determiners are simply a special subclass of Westerståhl's DET. The semantics of these new determiner expressions assumes the semantics of numerical expressions and their orderings as well as that of measurement functions (cf. section 2.2) in order to provide determiner specifications:

$$\begin{array}{l} \text{For } <\text{measure-determiner}>=<\text{measure-specifier}\, m>, \\ \quad \text{if } <\text{measure-specifier}> \text{ is `}>\text{', then} \\ \qquad [\![<\text{measure-determiner}>]\!] \;=\, \uparrow DET_{\{m' \mid m' > m\}} \\ \quad \text{, etc.} \end{array}$$

These notes are too incomplete to count as specifications, but they serve to indicate how the language and its model theory would be developed.

4.1 Logical Status of Measure Determiners

The present work has specifically linguistic aims: the account of what measure phrases mean, how they contribute to phrasal meanings, esp. NP meanings, and what follows semantically from them. There has been no attempt to identify which comparative statements (if any) should count as *logically true*. The present account assumes too much mathematics (the real numbers and their ordering, measure homomorphisms, etc.) to be regarded as a contribution to the semantic foundations of comparison.

5 Extensions

Our basic tack should by now be clear: a measure phrase specifies a set of measures from which a determiner meaning, in the sense of generalized quantifier theory, may be derived. The sections above show how a wide range of determiner meanings can be defined naturally on the basis of just such a set of measures.

We now turn to several interesting applications and extensions of the basic technique, viz. PARAMETRIC DETERMINERS in which a parameter appears as measure, and determiners derived from 3-PLACE RELATIONS on measures—the ADDITIVE and MULTIPLICATIVE comparative determiners.

5.1 Parametric Determiners

The only examples we've considered up to this point have been comparative determiners which specify measure sets absolutely, i.e., with respect to constant measures, e.g., *more than 3*. But the definitions of the determiners $\uparrow DET_M$, $\downarrow DET_M$, $\not\uparrow\not\downarrow DET_M$ don't depend on the parameters used to define M, i.e., all of the properties of the determiners are predictable even when parameters are used. For example, all of the following are well-defined:

more than n	$(> n)$	$\uparrow DET_{\{m \in \mathcal{M} \mid m > n\}}$
fewer than n	$(< n)$	$\downarrow DET_{\{m \in \mathcal{M} \mid m < n\}}$
exactly n	$(= n)$	$\not\uparrow\not\downarrow DET_{\{n\}}$

These definitions are useful in the analysis of constructions which Bresnan 1973 and others have termed "comparative subdeletion":

A saw more kids than B heard dogs

$$\exists m \in \mathcal{M} \; ((= m)\, x\, (\text{dog}(x),\ \text{hear}(b, x)),\ (> m)\, y\, (\text{child}(y),\ \text{see}(a, y)))$$

A saw fewer kids than B heard dogs

$$\exists m \in \mathcal{M} \; ((= m)\, x\, (\text{dog}(x),\ \text{hear}(b, x)),\ (< m)\, y\, (\text{child}(y),\ \text{see}(a, y)))$$

The formulas provide simple and correct renderings of semantics of the subdeletion cases—but it need not be that exactly these logical forms are used to render the readings (rather than some equivalent), nor that other forms must be less useful in providing a compositional account of the readings. In particular, we have not attempted to provide an account here of the mapping in case quantifiers appear in the *than* clause: *A saw more kids than everyone heard dogs.* See Pinkal 1989 for an account of these (compatible with this).

Remark: All the cardinality determiners treated up to this section have been first order, e.g., *more than 5*, *exactly 5*, and *fewer than 5*. The parametric determiners introduced here clearly go beyond first-order, however. For example, we can formulate the semantics of *most* using parametric determiners (cf. Barwise and Cooper 1981 for the proof that *most*—in the sense of "more than half"—is not first-order definable):

$$\begin{array}{lll} \mathrm{MOST}\, x(A(x),\, B(x)) & \Leftrightarrow & \exists m \in \mathcal{M} \quad ((=m)\; x\; (A(x),\, \neg B(x)), \\ & & \qquad\qquad (>m)\; y\; (A(y),\, B(y)) \end{array}$$

5.2 Additive Relations

Measure determiners are derived from the sets of measures; the latter have been specified above by 2-place relations on measures, especially '>' and '<'. But these specifications were chosen as introductory illustrations for their simplicity. 3-place relations on measures serve equally well to define measure sets, and some natural language constructions (illustrated below) provide excellent justification for exploiting this possibility. The additive and multiplicative conditions imposed on measure functions above (section 2.2) justify using addition and multiplication in the definition of relations on $\mathcal{M}$. We explore the additive relations in this section, the multiplicative ones in the next.

The additive relations seem best defined on the basis of the previous '>' and '<' relations, i.e.,

$$\text{for relation } R \text{ on } \mathcal{M},\, d \in \mathcal{M} \text{ let}(xRy,\ \triangle : d) \stackrel{def}{=} xRy \ \wedge\ |x-y| \ =\ d \qquad (4)$$

We borrow an idea from Situation Semantics here, where we use the rolename "$\triangle$" to designate an argument position rather than rely on order. This is not semantically different from order-based argument binding, but it is mnemonically easier. We discuss the motivation for the content of the definition below, but first we note the effect of the $\triangle$ specifications on measure set specification:

$$\begin{array}{lll} (x > y,\ \triangle : 2) & \Leftrightarrow & x > y \ \wedge\ |x-y| \ =\ 2 \\ & & x \ =\ y+2 \\ (x < y,\ \triangle : 2) & \Leftrightarrow & x < y \ \wedge\ |x-y| \ =\ 2 \\ & & x \ =\ y-2 \end{array}$$

and on derived determiner meanings:

(exactly) two more than n $\quad (>n, \triangle : 2) \quad \not\updownarrow\not\downarrow DET_{\{m \in \mathcal{M} | m = n+2\}}$

(exactly) two fewer than n $\quad (<n, \triangle : 2) \quad \not\updownarrow\not\downarrow DET_{\{m \in \mathcal{M} | m = n-2\}}$

In order to present the full range of natural examples, we borrow a device made popular by Situation Theory and Situation Semantics at this point, viz. that of RESTRICTED PARAMETER. This will allow us to represent the semantics of complicated comparisons in a relatively simple way. Barwise 1987 introduces restricted parameters as variables which are not yet bound by quantifiers (though they may eventually be), which are RESTRAINED to obey some restriction. For example, the sentence *a child walks* might be represented:

$$\mathrm{walk}(x|\mathrm{child}(x))$$

The semantics of restricted parameters requires that variable assignment functions satisfy the restriction associated with a given variable; in the example above, that $[\![x]\!]$ be found in $[\![\text{child}]\!]$. A formula containing a restricted parameter is satisfiable only if it is satisfied by some variable assignment which also satisfies all parameter restrictions.

We shall have occasion to parameterize 3-place relations both in the "pole of comparison" position as well as this new " $\triangle$" position. We show the effects of parameterizing measure set specifications in this argument position:

$$
\begin{array}{lcl}
(x > y,\ \triangle : (d|d \geq 2)) & \Leftrightarrow & x > y \ \wedge \ |x-y| \ = \ (d|d \geq 2) \\
 & & x \ \geq \ y+2 \\
(x < y,\ \triangle : (d|d \geq 2)) & \Leftrightarrow & x < y \ \wedge \ |x-y| \ = \ (d|d \geq 2) \\
 & & x \ \leq \ y-2
\end{array}
$$

The first of these formulas is thus for example true of triples x, y, d, where $x > y$ and $x - y = d$, AND, where d is additionally constrained to satisfy $d \geq 2$. Note that we could express this same meaning in a more complex fashion if we make use of λ-abstraction and explicitly existential quantification. For example, the first relation above could also be rendered:

$$\lambda x \lambda y (x > y \ \wedge \ \exists d (d \geq 2 \ \wedge \ |x-y| = d))$$

But we take the earlier representation to be more perspicuous.

From the parameterized set specifications, we can derive parameterized determiner meanings in the step that is by now familiar:

at least two more than n	$(> n, \triangle : (d\|d \geq 2))$	$\uparrow DET_{\{m \in \mathcal{M} \| m \geq n+2\}}$
at least two fewer than n	$(< n, \triangle : (d\|d \geq 2))$	$\downarrow DET_{\{m \in \mathcal{M} \| m \leq n-2\}}$

Finally, two natural language examples:

A taught exactly two more children than B trained dogs

$$
\begin{array}{ll}
\exists m \in \mathcal{M} & ((= m)\ x\ (\text{dog}(x),\ \text{train}(b,x)), \\
 & \quad (> m, \triangle : 2)\ y\ (\text{child}(y),\ \text{teach}(a,y)))
\end{array}
$$

A taught at least two more children than B trained dogs

$$
\begin{array}{ll}
\exists m \in \mathcal{M} & ((= m)\ \ x\ \ (\ \text{dog}(x)\ \ \text{train}(b,x)), \\
 & (> m, \triangle : (d|d \geq 2))\ \ y\ \ (\text{child}(y)\ \ \text{teach}(a,y)))
\end{array}
$$

We might have attempted other definitions of 3-place determiners:

$$
\begin{array}{lcl}
(x > y,\ \triangle : d) & \stackrel{def}{=} & x \ > \ (y+d) \\
(x < y,\ \triangle : d) & \stackrel{def}{=} & x \ < \ (y-d)
\end{array}
\qquad (5)
$$

The considerations that speak in favor of the former definition (4) are not conclusive, but they may be worth review. To begin, I assume that specifications *exactly* in *exactly two more* etc. modify the the difference ($\triangle$) rather than anything else. There is good syntactic reason for this:

- the specifications appear only with the differences
  - *exactly/at least two more ...*

– **exactly/at least ∅ more ...*

- the specified differences may be coordinated
  – *at least two and perhaps even ten more ...*
- the specified differences may be used to answer questions
  – *How many more ...?*
  – *Exactly/At least two.*

Compositionality then argues that the syntactic constituent be interpreted as a semantic unit as well. Once this is accepted, then the definition (4) is to be preferred to the alternative (5).

- the latter definitions would not distinguish *exactly two more* from *at least two more*; the former does.
- the $(> n, \triangle : \delta)$ relation in (4) can never hold of $n' < n$, since it is required that $n' > n \wedge |n - n'| = \delta$; the alternative is more lenient in some cases:

$$(> n, \triangle : (d|d \leq 2))$$

Alternative (5) reduces this to $(n' < n + 2)$, and this seems too lenient. The distinction is relevant in sentences such as the following,

Smith hired at most two consultants more than Brown

Definition (4) makes this false if Smith hired fewer consultants than Brown; definition (5) makes it true. The former seems correct.

Finally, the example above also highlights a complication caused by the possibility of parameterizing the specifications of differences, which is that the monotonicity properties of measure sets cannot be "read off" the relation constant used to specify them. This will complicate clauses in the model theory (for 3-place determiners) for LMD. Cf. Section 4 above.

5.3 Multiplicative Relations

The additive condition imposed on μ justifies using **multiplication** in the definition of relations on $\mathcal{M}$. This section is parallel to section 5.2, only somewhat more abbreviated. We concentrate on the semantics underlying the factors used in so-called "equative" comparison, summarizing the differences between these and other comparatives below. We again define the 3-place multiplicative relations much as we defined the additive ones. The choice of relational symbol ('=') is motivated here only by the linguistic tradition of referring to the typical uses of multiplicative comparisons as EQUATIVE. Note that it plays no role in the definiens.

$$\text{For the relation} = \text{on } \mathcal{M},\ f \in \mathcal{R} \text{ let } (x = y,\ * : f) \stackrel{def}{=} x/y = f$$

We turn some examples of multiplicatively specified measure sets:

$$\begin{array}{rcl} (x = y,\ * : 2) & \Leftrightarrow & x/y = 2 \\ & & x = 2 \cdot y \\ (x = y,\ * : 1/2) & \Leftrightarrow & x/y = 1/2 \\ & & x = y/2 \end{array}$$

And some natural language examples:

(exactly) twice as many as n	$(= n, * : 2)$	$\not\uparrow\not\downarrow DET_{\{m \in \mathcal{M} \mid m = 2 \cdot n\}}$
(exactly) half as many as n	$(= n, * : 1/2)$	$\not\uparrow\not\downarrow DET_{\{m \in \mathcal{M} \mid m = n/2\}}$

Factor specifications may be parameterized as well:

$$\begin{array}{rcl} (x = y,\ * : (f|f \geq 2)) & \Leftrightarrow & x/y \ = \ (f|f \geq 2) \\ & & x \ \geq \ 2 \cdot y \\ (x = y,\ * : (f|f \geq 1/2)) & \Leftrightarrow & x/y \ = \ (f|f \geq 1/2) \\ & & x \ \geq \ y/2 \end{array}$$

which leads to a treatment of determiners such as the following:

at least twice as many as n $\quad (= n, * : (f|f \geq 2)) \quad \uparrow DET_{\{m \in \mathcal{M} | m \geq 2 \cdot n\}}$

at least half as many as n $\quad (= n, * : (f|f \geq 1/2)) \quad \uparrow DET_{\{m \in \mathcal{M} | m \geq n/2\}}$

Finally, we present two example translations:

A taught exactly twice as many children as B trained dogs

$$\begin{array}{ll} \exists m \in \mathcal{M} & ((= m) \ x \ (dog(x) \, , \, train(b,x)), \\ & (= m, * : 2) \ y \ (child(y) \, , \, teach(a,y))) \end{array}$$

A taught at least half as many children as B trained dogs

$$\begin{array}{ll} \exists m \in \mathcal{M} & ((= m) \ x \ (dog(x) \, , \, train(b,x)) \\ & (= m, * : (f|f \geq 1/2)) \ y \ (child(y) \, , \, teach(a,y))) \end{array}$$

Just as in the case of the additive determiners, other definitions of 3-place multiplicative determiners are also available. And it will be noted that the definition proposed here for multiplicative determiners isn't even parallel to (4), the definition proposed for additive determiners. In particular the inference to the equality WITHOUT the factor specification has to be invalid:

$$\begin{array}{l} (x > y,\ \triangle : d) \ \Rightarrow \ x \ > \ y \\ (x = y,\ * : f) \ \not\Rightarrow \ x \ < \ y \end{array}$$

Cf. the case of $f = 1/2$ above.

Other differences are even more striking when we examine factor specification in combination, not with equatives, but rather with comparatives. These differences are summarized in the table below.

Type	**Factor**	**Example**	**Proportion**	**Formula**
$=$	> 1	three times as much	$x/y = 3$	$x/y = f$
	< 1	one-third as much	$x/y = 1/3$	$x/y = f$
$>$	> 1	three times more	$x/y = 3$	$x/y = f$
	< 1	one-third more	$x > y \wedge$ $\lvert x - y \rvert / x = 1/3$	$x/y = 1 + f$
$<$	> 1	three times less	$x/y = 1/3$	$x/y = 1/f$
	< 1	one-third less	$x < y \wedge$ $\lvert x - y \rvert / x = 1/3$	$x/y = 1 - f$

The table above is restricted to the mass determiners. Plural determiners substitute *many* for *much*, and *fewer* for *less*, not always felicitously.

It's worth noting that the FRACTIONAL specifiers (in the fourth and the sixth lines) might also be grouped with the additive determiners, since both addition and multiplication are involved. There is obviously more investigation to do here. Among other topics, it would be worth checking which of the combinations make semantic sense, since not all combinations are felicitous, and some infelicities may be semantic, rather than purely syntactic. For example, the multiplicative specifiers are peculiar when used in combination with plural $\downarrow -DET$'s:

∗ . . . three times fewer . . .
∗ . . . one-third fewer . . .

(But note that *three times more* and *one-third more* are heard frequently, both for mass and for plural determination.) The peculiarity extends imperfectly to the prescriptively preferred equative form, including surprisingly the mass determiners:

∗ . . . three times as few . . .
∗ . . . one-third as few . . .
∗ . . . three times as little . . .
∗ . . . one-third as little . . .

5.4 Noncomparative Measure Determiners

Measure determiners can arise from ANY specification of a set of measures–the specification needn't be based on comparatives, as several of the examples above show. One of the most interesting classes of noncomparative measure determiners uses simple measure phrases as determiners, for example:

Three people have arrived.
Two kilos of cocaine were seized.

There's a longstanding debate about whether sentences such as these should be analyzed as meaning e.g. *At least three people have arrived* or *Exactly three people have arrived*. On the one hand, there are situations in which such sentences appear to be used to assert the weaker meaning:

If three people have arrived, we can play. Have three people arrived?
—Yes, three people have arrived. In fact, five people have arrived.

Furthermore, there is a reasonable Gricean account, due to Horn 1972, of how the stronger ("exactly") readings might be inferred from the weaker ("at least") readings. This account postulates that a speaker is normally as informative as possible, so that he therefore uses the more informative (larger) measure specification where possible.

On the other hand, not all uses of the simple measure specifications are compatible with the postulate of weaker meaning.[9]

The present work cannot decide this question (though it should be clear that both meanings are readily formulated in LMD), but we'd like to contribute one point to the debate, viz. that, at the level of compositional semantics—as opposed to the level of sentence meaning, there is no very satisfying locus for the "Gricean" sort of meaning. The are two likely candidates for such a locus, the number (or measure phrase) itself, and the relation to which it supplies

9. Jonathan Ginzburg discussed the following example, in a talk "Informativeness Evaluated" to the Situation Semantics Working Group at CSLI in Winter, 1990. He credited unpublished work of Carston 1985:

If you eat 1500 calories a day, you'll lose weight.

an argument, in this case the specification of the measure set. If the meaning of *four* were '≥ 4', then the meaning of all the measure specifiers *at least, exactly, ...* and all the expressions of relations between them would be extremely counterintuitive. If on the other hand, the relations to which measures supply arguments all have a built-in bias up the scale, then there is absolutely no way of deriving the meanings of phrases with specified measures. To see this, assume that *d more than m* means ***at least d*** *more than m*, and now try to feed a specified measure into that argument position, e.g. *at most d'*.[10]

It would therefore seem that more sophisticated treatments of quantity implicatures are required. A more promising tack might be to view unspecified measure phrases as implicitly requiring a specification from which a measure set can be derived. The specification is often provided explicitly (*at least,* etc.), but in the absence of explicit specification, an appropriate candidate must be found (perhaps by default). But pursuing this topic here would bring us too far afield.

6 Implementation

The work described in this paper was implemented as an extension to $\mathcal{NLL}$, a meaning representation language built on GQT (although only a restricted class of multiplicative quantifiers was included). In addition to the meaning representation described above, one rule of inference specific to comparatives was implemented; this rule exploits the transitivity of the order relations on measures.

$\mathcal{NLL}$ proceeds from a core consisting of the language of generalized quantifiers (with only atomic determiners) to a set of extensions which are intended to allow experimentation with various approaches to natural meaning representation, inference, and application-interfaces. The core together with the extensions thus comprises a possibly incompatible set of logical languages. $\mathcal{NLL}$ and its implementation is described in more detail in Nerbonne et al. 1993 and Laubsch 1989.

The original implementation was carried out in the Refine language, chosen because it provides (i) a grammar facility for language definition, including parser generator and printer; (ii) facilities for transformation either at the level of syntactic expression (in the defined language) or at the level of data structure; (iii) many high-level programming constructs (sets, mappings, etc.) which ease coding; and (iv) some support for the concept of language extension through grammar inheritance. Refine generates Common Lisp programs, and the entire system was integrated into HP-NL, a natural language processing program developed at Hewlett-Packard Laboratories and written in Common Lisp (cf. Nerbonne and Proudian 1987).

7 Conclusions and Prospects

An extension of this approach to the ternary determiners discussed by Keenan and Moss 1984 would be interesting, since these used the same additive and multiplicative properties of measures exploited here:

Smith hired three more men than women
Smith hired three times as many men as women

10. The same point can be made against the proposal to take adjectival meanings such as *t tall* to mean *at least t tall.* Furthermore, the *less* variant of the comparative is also impossible to interpret semantically once one assumes that the base (positive) adjective denotes a relation between objects and measures that they are taller than.

Another obvious application of the approach using measure theory is to adjectival comparison; this would resemble the approach in Cresswell 1976.

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