On the Semantics of Comparison

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1 Introduction

Our Paper presents an abridged version of Cresswell’s The Semantics of Degree (Henceforth /SD/) which we regard as an enlightening and fundamentally sound semantic treatment of comparatives. We will propose an extension of his analysis to account for “mixed comparatives” such as

(1a) Ophelia is more beautiful than intelligent
(1b) Bill is taller than Tom is clever

These sentences are explicitly excluded from Cresswell’s treatment. We hold that a great number of examples of this type are perfectly natural. In other words, Cresswell’s notion of comparison is not general enough in this respect. The price we pay for this extension is the loss of generality in another respect. One of the major virtues of the analysis proposed in /SD/ is that it provides a unified treatment of comparison involving adjectives, mass-nouns and plurals. We maintain on the contrary that comparison as we hope to elucidate below has different properties according to the kind of terms that are involved. That is why we opt for not treating adjectival, massnour and plural-comparison on a par.

Our proposal relinquishes some of the simplicity and elegance of Cresswell’s original formulation; but the fact that we attempt to cover more ground may be well worth the sacrifice.
II \textbf{\textlambda\text{-Categorial Languages}}

In this section we do not intend to give all the formal definitions, but will have to rely on the reader’s familiarity with the mechanisms of a \textlambda\text{-categorial language or at least on his willingness to look up some of the definitions in /SD/}. The first draft of this paper gave all the formalism and was much more rigorous. We hope that ‘deformalisation’ has made it more readable and has not affected clarity. Let us mention only some bare necessities:

We first need the notion of a SYNTACTIC CATEGORY. For our purposes the basic categories are 0 and 1, the categories of SENTENCE and NAME respectively. Given expressions of categories $\tau$, $\sigma_1$,...,$\sigma_n$ the functor which takes expressions of categories $\sigma_1$,...,$\sigma_n$ to form an expression of category $\tau$ is of category $\langle \tau, \sigma_1, ..., \sigma_n \rangle$.

How expressions from different categories can be combined, sometimes with the help of the \textlambda-operator, can be found in /SD/ p. 262.

We take the liberty of using some more manipulative syntactic devices as well. In particular, we will assume without discussing devices for reordering syntactic expressions and, in at least one case (viz. \textit{pos}) we will introduce a symbol into the formal language which does not appear in the surface of English. Then we must assume that we can “filter” this element out of a later syntactic string.

The semantics is defined as usual, i.e. there is a function $D$ which associates with each syntactic category $\sigma$ the set $D_\sigma$ of possible semantic values for elements of $\sigma$. $D_0$ is then the set of propositions and $D_1$ the set of individuals. The elements of $D_0$ are regarded as subsets of $W$, the set of possible worlds.

We assume intransitive verbs and common nouns to be 1-place predicates, i.e. of category $\langle 0, 1 \rangle$. (We dispense with a syntactic distinction here, noting that one is possible$^9$). Transitive verbs are of category $\langle 0, 1, 1 \rangle$. The interpretation of the symbol \textit{man} of category $\langle 0, 1 \rangle$ for example is:

$V(\textit{man})$ is the function $\omega \in D_{\langle 0, 1 \rangle}$ such that for any $a \in D_1$, $a$ is in the domain of $\omega$ iff $a$ is a physical object in some $w \in W$ and for any $w \in W$, $w \in \omega(a)$ iff $a$ is a man in $w$.$^5$

In this semantics we do not make use of points of time or contextual indices, so that it will be inadequate for the treatment of many expressions of natural language. A possible-world semantics without reference to points of time is, however, sufficient to deal with many of the problems we are interested in.

III \textbf{Adjectives and Comparison}

CRESSWELL agrees with MONTAGUE and PARSONS in that he takes the attributive use of adjectives to be prior to their predicative use$^9$. Adjectives are therefore interpreted as modifiers of nouns. This suggests the categorization $\langle \langle 0, 1 \rangle, \langle 0, 1 \rangle \rangle$, that is, \textit{tall} being of category $\langle \langle 0, 1 \rangle, \langle 0, 1 \rangle \rangle$ takes the common noun \textit{man}.
of category \(\langle 0,1 \rangle\) to form the complex common noun phrase *tall man* of category \(\langle 0,1 \rangle\). Following CRESSWELL we will call expressions of category \(\langle \langle 0,1 \rangle, \langle 0,1 \rangle \rangle\) ordinary noun modifiers.\(^7\) This categorization, however, proves difficult to incorporate into a theory of comparative adjectives.

The refined analysis CRESSWELL proposes rests on the notion of "points on a scale" which we might have in mind when we make a comparison. The scale can be formally represented by a relation and the points on the scale by the field of that relation. If \(\langle \rangle\) is an order relation which is transitive and anti-symmetrical\(^8\) and if \(\mathcal{F}(\langle \rangle)\) denotes the field of the relation, we define:

A *degree* (of comparison) is a pair \(\langle u, \rangle\) where \(\langle \rangle\) is a relation and \(u \in \mathcal{F}(\langle \rangle)\), (\cite{SD}, p. 266).

We now assume that the phrase *x-much tall man* underlies the phrase *tall man*. That means: \(\langle \text{tall, man} \rangle\) is a two place predicate with roughly the meaning of \(y\) is a man who is tall to degree \(x\) and tall is of category \(\langle \langle 0,1 \rangle, \langle 0,1 \rangle \rangle\). Its semantics is as follows:

\(V(\langle \text{tall} \rangle)\) is a function \(\xi \in D_{\langle \langle 0,1,1 \rangle, \langle 0,1 \rangle \rangle}\) such that where \(\omega \in D_{\langle 0,1 \rangle}\), \(\omega\) is in the domain of \(\xi\) iff \(\omega\) is a property whose domain contains only physical objects. For any \(a, b \in D_{1}\) in the domain of \(\xi(\omega)\), and any \(w \in W\), \(w \in \xi(\omega)(a, b)\) iff \(w \in \omega(a)\) and \(b = \langle u, \rangle\), where \(\rangle\) is the relation whose field is the set of all \(v\) such that \(v\) is a spatial distance, and \(\langle v_1, v_2 \rangle \in \rangle\) iff \(v_1\) is a greater distance than \(v_2\), and \(u\) is the distance between \(a\)'s extremities in \(w\), and in the case of most \(c\)'s such that \(w \in \omega(c)\) this distance will typically be vertical. (\cite{SD}, p. 267)

Before discussing the comparative case we present CRESSWELL's treatment of the positive. In order to capture

\[(2)\quad\text{Bill is a tall man}\]

we have to get rid of the degree-argument in *tall*. To this end CRESSWELL introduces a symbol *pos* into the \(\lambda\)-categorial base which does not appear on the surface. *pos* makes an ordinary noun modifier out of a modifier like *tall*, i.e. *pos* of category \(\langle \langle 0,1 \rangle, \langle 0,1 \rangle \rangle\) takes *tall* of category \(\langle \langle 0,1 \rangle, \langle 0,1 \rangle \rangle\) to form the phrase \(\langle \text{pos, tall} \rangle\) of category \(\langle 0,1 \rangle, \langle 0,1 \rangle \rangle\). Semantically the idea is to interpret a sentence such as (2) as something like *Bill is taller than the average man*, i.e. to base the positive on the comparative.

Given the semantics of *pos* \(V(\text{pos})\) cf. /SD/, p. 272), we have:

\[(2)\quad\text{is true iff Bill is a man and there is a degree } b = \langle u, \rangle \text{ which can be attributed to Bill's tallness and } u \text{ is towards the top of the scale determined by } \rangle_{\text{tall}} \text{ when compared to those degrees } a = \langle v, \rangle \text{ of tallness which occur as tallness degrees of men.}\]

The last clause of the definition restricts the comparison to relevant objects. It might be rephrased as: \(u\) is toward the top of the scale when restricted to those \(v\) such that for some \(c\), \(c\) is a \(v\)-much tall man. Thus for the sentence *Bill is a tall man* to be true, we require not that Bill be a tall entity, but that Bill be tall as compared to the class of men.
We will now introduce the comparative symbols *er than* and *as..as* of category \(\langle 0, (0,1), \langle 0,1 \rangle \rangle\):\(^9\)

\(V\ (*er than*)\) and \(V\ (*as..as*)\) are functions which compare degrees, or more correctly, they are functions whose *arguments* are functions which take degrees as arguments.\(^{10}\)

In CRESSWELL's semantics these functions can only operate on degrees of the same scale. If \(a = \langle u, >_1 \rangle\) and \(b = \langle v, >_2 \rangle\) and \(>_1 \not\supset >_2\) comparison is not possible. A unique relation ‘\(>\)’ is required so that \(a = \langle u, > \rangle\) and \(b = \langle v, > \rangle\).

*John is taller than Bill* is true iff \(u > v\) holds for the degrees \(a\) and \(b\) of tallness (\(a\) and \(b\) as above), which can be attributed to John and Bill respectively.

\(V\ (*as..as*)\) works similarly, only we require \(u > v\) or \(u = v\), if the conditions are as above in other respects.

As these definitions imply that we can only compare within one scale, a sentence involving “mixed comparison” such as

(3) Ophelia is more beautiful than intelligent

is not defined, *long* and *tall*, however, are conceived as defining the same relation and the same scale, so that

(4) Ophidia is a longer snake than Bill is a tall man

can be interpreted semantically. Our extension of CRESSWELL's analysis is designed to provide a semantics for sentences such as (3). But before going into this question, we shall exhibit CRESSWELL's treatment more completely.

Since we are not concerned with and do not have anything to add to CRESSWELL's treatment of the superlative, we do not discuss it here.

In order to explain how CRESSWELL distinguishes between (3) and (4) we have to discuss the notions “scale” and “degree” in more detail. As noted above (sentence (4)) we can compare *long* and *tall* because they share a scale. *tall* and *short* on the other hand are associated with different scales:

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        ↑
        ↓
    1     2      increasing tallness
    ↓     ↑
  2     1      increasing shortness
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This explains why we find (5) peculiar

(5) ?Bill is taller than Sam is short\(^{11}\)

since the scales associated with *tall* and *short* are directed differently, even if they are measured similarly. There is a snag here, however, since we do use comparisons between short and tall. Sentence (6) is well-formed.

(6) Sam is taller than five feet but shorter than six feet
CRESSWELL regards the pattern in (7) as further evidence that tall and short are bound to at least different relations:

(7)  
Sam is six feet tall  
?Sam is six feet short (/SD/, p. 274)

which leads to the definition of six feet as a degree on the tallness scale. We attach little credence to this argument since (8) is also well-formed:

(8)  
Sam is two inches shorter than Bill

But, since we have no suggestion for improvement in (7), we will not alter the definition of ‘degree’. Like CRESSWELL, we will take degrees to be pairs consisting of a point and the relation involved; we will, however, construct various types of scales to account for the other examples.

Further adjustments are prompted in considering comparisons between two different scales. Sentences like

(9)  
I am more honest than you are mad  
Joan is uglier than Ann is pretty  
Joan is smarter than Ann is pretty  
Bob is more intelligent than Jack is stupid  
Bill is more handsome than intelligent  
Mary is as attractive as she is intelligent  
Tom is as annoying as he is persistent

are accepted by most speakers and strongly suggest that we do compare different scales. We will try to capture the “mixed” comparison in (9) through the mechanism of mapping one scale to another, which will be discussed at length further below. We turn immediately to a discussion of the make-up of the domains of these mappings, i.e. the scales we are going to use.

Measuring adjectives, such as long, tall and heavy always seem to suggest numbers on a scale, which reduces the problem of choosing an appropriate scale. Other adjectives, e.g. beautiful, admit no such straightforward metrical interpretation. We will follow CRESSWELL, who suggests that one use classes of objects of equal beauty as degrees on the beauty scale.

More formally: We take the comparative of an adjective A, A-er than, as in some way defining an order relation $\Phi_A$ on the set of objects. This relation can be used to define an equivalence relation $R$, which in turn defines equivalence classes. These resulting classes become the elements of the field of the constructed order-relation $\succ \Phi_A$.

A degree is now a pair $(u, \succ)$, where $u$ is an equivalence class and $\succ$ a relation $\succ \Phi_A$. These definitions are general enough to include paradigm examples of scales, such as the metric system of weights and measures, but they also allow us to speak of scales associated with concepts like beauty.
A problem arises in that an “average” is impossible to determine without knowing all the classes which contain a relevant element and the number of relevant elements in these classes (for example, the number of men in the class of *six foot tall men*).

It certainly wouldn’t be surprising if context played a considerable role. ‘tall’ is obviously different in contexts like basketball games.\(^{15}\)

We want to introduce a new kind of scale. We maintain first that antonymous adjectives share a scale.

Supporting this we have the sentences:

(6) Sam is taller than five feet but shorter than six feet
(10a) X is bigger than Y entails that Y is smaller than X
(10b) This stone is bigger than that one entails that that stone is smaller than this one.

We define as one sort of scale (type A): when the comparative-relations of both relations of both adjectives of a pair of antonyms have the same field, and they are inverse relations of each other, as e.g. *big-small, tall-short,* and *high-low:*

\[
\begin{array}{c}
| & a & b & \rightarrow & a < b \text{ iff } b > a \\
\hline
\end{array}
\]

This will account for the entailments in (10) and the possibility of (6). We note, however, that the pattern in (10) isn’t universal in antonymous adjective pairs:

(11) ? Joan is more beautiful than Agnes entails that Agnes is uglier than Joan.
    ?Richard is more arrogant than Sam entails that Sam is more unassuming than Richard.
    ?Tom is more obnoxious than Peter entails that Peter is more pleasant than Tom.

In fact, these adjectives are different in several semantic aspects:

(12) ?Joan is more beautiful than Arabella but both are ugly
    ?Richard is more arrogant than Sam but both are rather unassuming.
    ?Tom is more obnoxious than Peter but both are rather pleasant.

This set of sentences suggests a second sort of scale for pairs like *beautiful-ugly* as opposed to *big-small.* We define then as a second sort of scale (type B): when the fields of the comparative relations of the paired adjectives do not overlap, but they are conceived nonetheless as forming one scale, as e.g. *beautiful-ugly, polite-rude* and *arrogant-unassuming:*

\[
\begin{array}{c}
\leftarrow & \cdots & \cdots & \rightarrow \\
\end{array}
\]

This sketch is meant to suggest that while it is intuitively plausible to assign e.g. a ‘degree of beauty’ to all individuals, this possibility isn’t realized in the language; thus the ‘gap’ in the line. We might regard those individuals with e.g. degrees of
politeness-rudeness falling in the area of the dotted line as failing to meet the minimal requirements to count as either polite or rude. In this sense, these adjectives involve a ‘norm’, indicated by the vertical lines.

To mark the different scales and thereby distinguish the classes of adjectives, we introduce two types of relations, ‘>’ and ‘>n’ (‘n’ for ‘norm’), corresponding respectively to types A and B above.

For type ‘>’ relations we roughly have: The construct relation $\succ_{\Phi_A}$, operates on the same field as the construct relation $\prec_{\Phi_{A_2}} = A_2$ being the antonym of $A_1$. A type ‘>n’ relation is built up from the fields of the two construct relations of antonymous adjectives such that the fields do not overlap, but the elements of the fields – i.e. the equivalence classes – are defined via one common quality.

In our semantics of tall, short, beautiful and ugly we will then have to make clear what kind of relation is involved. In the case of $V(tall)$ we then have to insert the clause: $\succ_{\text{taller}}$ is a relation of type ‘>’; except for this insertion, $V(tall)$ reads as before.

For $V(short)$ we now have to specify that though $\prec_{\text{shorter}}$ ranges over the entire scale, short does not; it is confined to the lower parts of the shared scale, because it involves a norm, or in other words it is marked.

The semantics of beautiful and ugly reads as follows:

$V(\text{beautiful})^16$ is the function $\xi \in D_{(0,1),(0,1)}$ such that where $\omega \in D_{(0,1)}$, $\omega$ is in the domain of $\xi$ iff $\omega$ is a property whose domain consists only of perceptual objects and for any $a, b \in D_1$ in the domain of $\xi(\omega)$ and any $w \in W$, $w \in \xi(\omega)$ (a, b) iff $w \in \omega(a)$ and $b = \omega_1, \succ_{\Phi_1}$, where $\succ_{\Phi_1}$ is a relation of type ‘>n’.

The field of this relation consists of equivalence classes whose elements are assigned to the class on the basis of form characteristics or combinations thereof.

$\langle v_1, v_2 \rangle \succ_{\Phi} \text{iff } v_1, v_2 \in \mathcal{F}(\succ_{\Phi})$ and for the representatives (i.e. random elements) of the classes, $v_1, v_2$ the following holds: according to a standard based on a social and cultural consensus $v_1$ and $v_2$ are both regarded as aesthetically pleasing, but $v_1$ more than $v_2$.

$V(\text{ugly})$ also involves a type ‘>n’ relation $\prec_{\Phi_2}$, and we would define:

$\langle v_1, v_2 \rangle \prec_{\Phi} \text{iff } v_1, v_2 \in \mathcal{F}(\prec_{\Phi})$ and for the representatives of the classes, $v_1$ and $v_2$, the following holds: $v_1$ and $v_2$ are both aesthetically displeasing, but $v_1$ more than $v_2$.

$\succ_{\Phi_1}$ taken together with $\prec_{\Phi_2}$ forms a complete scale of type B.

Both definitions, $V(\text{beautiful})$ and $V(\text{ugly})$, are so constructed so that for both adjectives A, $\text{x-much} A$ (intuitively) implies A.

This and the relation type ‘>n’ accounts for the peculiarity of sentences (11) and (12) above.

Compare also the sketch, p. 6. For the same reason, we will have to alter the definition of pos; since in certain cases (viz. those involving type B adjectives) we no longer have to mention “toward the top of the scale”.

7
Our treatment of *er than* will include cases like:

(13) Bill is taller than Ophidia is long.  
    Bill is taller than Bob.
(14) Bill is taller than Bob is strong  
    Bill is more handsome than intelligent

The first example in (14) illustrates a comparison between two scales of type A, ’>’, and the second example is a comparison between two scales of type B, ’>_n’.

We picture these comparisons as effected via a mapping from the scales associated with the involved adjectives to a third scale of the same type.

A further possibility is that the compared adjectives are of different types:

(15a) Bill is more handsome than tall  
(15b) This cigarette is longer than it is good-tasting

In trying to capture the mechanism of comparison here, we follow SCHOPF, 1969, who suggests that the norms associated with *handsome* and *good-tasting* are ‘grafted onto’ *tall* and *strong*. The hypothesis would be that what we compare are the deviations from the respective norms. Thus we understand (15a) as asserting that the more unusual or more striking characteristic is Bill’s handsomeness rather than his height. In our semantics, this means that we have to map both scales onto a scale of type ’>_n’ and compare two points on the new scale. These cases demand a fairly complicated treatment of *er than* because of the different mappings required.

We have to distinguish:

a) >\Phi_1 = >\Phi_2 and *er than* defined as before.

b) >\Phi_1 \neq >\Phi_2 and bi) both of the same type.

or bi) of different types.

For type bi) the third scale is always of type ’>_n’ because we ‘graft the norm’. In such a comparison both adjectives behave as if normed, so we map both to a scale with a ‘norm’.

There are a few minimal conditions for such a mapping: it must be order preserving; the order relation on the third scale has to be an expansion of both image relations of the original scales. What is meant by this can best understood by looking at sketch bi). 17)

Because all this looks rather complicated and for a couple of other reasons we propose another analysis in part VII. In many respects the modifications suggested there will simplify the treatment.

The following sketches may illustrate the different cases:

for a)  

\[ \text{shorter, taller} \quad a < b, \quad b > a \]
As we pointed out before we now have to redefine $V(pos)$, distinguishing the way it works on adjectives with type $'>'$ or $'>_n'$ relations. For type $'>_n'$ we no longer have to mention 'towards the top of the scale'.

$V(as...as)$ may be defined as before. We only have to note that the comparison may also be carried out via the third scale constructed here again as in $V(er\ than)$.

This treatment provides a semantics for sentences like:

(16a) John is tall
(16b) Arabella is beautiful
(16c) John is six feet tall
(16d) John is taller than Arabella
(16e) Arabella is taller than intelligent
(16f) John is as tall as Arabella
(16g) John is as handsome as tall.

Because the mappings we have employed are not precisely defined, the truth-conditions for sentences (16a)—(16g) inherit a certain amount of vagueness. But it seems to us that there are no sharp, generally agreed upon truth conditions for comparisons such as (16d) and (16g), an opinion which is supported by the disagreement of speakers in the interpretation of such sentences. Most speakers seem to agree, however, that comparison is possible in cases like the above, supporting the existence for the proposed mappings.

The question of whether or not to regard sentences like (16e) and (16g) as having a definite semantic value shouldn’t rest solely on our immediate intuitions about its truth conditions. The relation of consequence e.g. relies on the definition of semantic value. Thus if we define:
A formula \( \alpha \) is a consequence of a set \( A \) of formulae (in symbols: \( A \models \alpha \)) iff for all models \( M \), if \( M \models A \), so \( M \models \alpha \).

(or, in CRESSWELL's notation:

\[ \text{... iff for all value assignments } V, \bigcap_{\beta \in A} V^*(\beta) \subseteq V^*(\alpha'). \]

It is important not to be careless about specifying '\( M \models \alpha' \) or \( V^*(\varphi) \) at crucial points. Since there is no \( M \) such that \( M \models \alpha \), where \( \alpha \) is sentence (16e), everything would seem to follow from (16e), which certainly isn't so. Nor will it do to add some sort of special clause, essentially excluding (16e) from inclusion in the consequence relation. Compare the following set of sentences (where a. and b. are premises and c. is the conclusion):

(16')

a. Ophelia is more beautiful than intelligent
b. Ophelia is more intelligent than Jane
c. Ophelia is more beautiful than Jane is intelligent

Our proposed extension of CRESSWELL's semantics will be able to account for the validity of (16'). As we noted, (16') is trivially valid in CRESSWELL's system, since there is no model for (16') a. Counterexamples are easy to construct. Thus (16'') is also valid in CRESSWELL's treatment (where a., b., c. as before):

(16'')

a. Ophelia is more beautiful than intelligent
b. Ophelia is more intelligent than Jane
c. Jane is more intelligent than Ophelia is beautiful

For these rather general reasons as well, we prefer the redefinition of er than proposed above.

IV Mass — Nouns

In the following two sections we will discuss CRESSWELL's remarks on mass nouns and the plural.\(^{18}\) Neither of these topics is treated at length in CRESSWELL's paper and in neither case will we attempt a complete criticism. We will aim mainly to indicate some problem areas in CRESSWELL's suggested treatment, and, where possible, suggest how solutions might be found.

CRESSWELL analyzes mass-nouns much as he does comparative adjectives. Thus \( \text{water} \) is a two place predicate with a meaning something like:

(17) \( x \) is an amount of water of volume \( y \)\(^{19}\)

To analyse

(18) More water ebds than mud flows

CRESSWELL first introduces a function \( \text{tot} \) out of category \( \langle \langle 0,1 \rangle, \langle 0,1,1 \rangle, \langle 0,1 \rangle \rangle \), which (intuitively) is applied to arguments like \( \text{water} \) and \( \text{ebbs} \) to yield a function
in \((0,1)\) whose domain consists of degrees and which is satisfied by the degree of the largest volume satisfying both water and ebbs. Thus \((\text{tot}, \text{water}, \text{ebbs})\) will be true exactly for that degree \((u, >)\), where \(u\) is the volume of ebbing water and \(>\) is the greater than relation with respect to volumes. In the semantics of (18) it is then a simple matter to apply \(er\) \(than\) to the two degrees which make these \(\text{tot} - \) functions true. \((\langle \text{tot}, \text{water}, \text{ebbs} \rangle \ er \ than \langle \text{tot}, \text{mud}, \text{flows}\rangle)\) will be true just in case the volume of ebbing water exceeds the volume of flowing mud.

deg is the counterpart to pos in the treatment of mass nouns.\(^{20}\) Thus, if \((\langle \text{pos, tall}, \text{man} \rangle)\) is satisfied by objects in the class of men whose height is ‘considerably toward the top’, \((\text{deg, water, ebbs})\) will be satisfied by objects fulfilling similar conditions. CRESSWELL defines \(\text{deg}\) out of \((0, \langle 0,1,1 \rangle, \langle 0,1 \rangle)\) and paraphrases his definition (with reference to this example) in the following way:

“(\(\text{deg, water, ebbs}\)) will be true iff the total volume of ebbing water is considerably toward the top end of the greater than scale when this scale is restricted to volumes of water in \(w\).”\(^{21}\)

CRESSWELL intends the definition of \(\text{deg}\) to provide an analysis of both sentences in (19):

(19a) Much water ebbs\(^{22}\)
(19b) Water ebbs

He regards \(\text{much}\) then as an optional surface realization of \(\text{deg}\).\(^{23}\) It is not clear to us that (19b) can be understood as sharing any set of truth conditions with (19a). Indeed, under its most natural, viz. generic reading (19b) may be true at \(w\) even if there is no ebbing water at \(w\). We do not intend to analyse the generic meaning of (19b).

We return now to the discussion of \(\text{deg}\) as an analysis of (19). The definition of \(V(\text{deg})\), as presently formulated, would stipulate that the greatest volume of ebbing water is ‘much’in \(w\) iff it is considerably large in comparison to all other volumes of water in \(w\). Consider now the sentence

(20) Sam drinks much water

where, for the sake of simplicity, \(\text{Sam drinks } x\) is analyzed as an expression of category \((0,1)\). This would be (intuitively) true, if, say, Sam drank 10 liters of water. To get the semantics right here, we would have to assume that for a certain class of reference points 10 liters is a considerably large volume of water. But now consider the situation where Sam drinks his 10 liters a) on the bank of a river or b) at the beach:

(21a) Much water flows
(21b) Much water ebbs

Here we are forced to the result that, if (20) is true then the sentences in (21) are true, as long as 10 liters of water are a) flowing or b) ebbing.
These examples suggest that the notion of *much* cannot be made precise solely on the basis of comparison to the volumes of stuff denoted by mass nouns. That is, we don't say that *much water ebbs* because the volume of ebbing water is a large volume of water, but because it is a comparatively large volume of ebbing water.

We seem to need some sort of modal notion here. That is, a volume is *much* if it is large in comparison to what it could be, or what one would expect it to be. We can do this by comparing the relevant volumes of water to similar volumes of water in other possible worlds. We may wish to compare it to similar volumes in all possible worlds, or we may compare it only to volumes in worlds standing in a particular accessibility relation 'R' to the one under consideration. We might wish for example to consider only those possible worlds compatible with our expectations. The various accessibility relations may provide a more direct method of interpreting multidimensional adjectives (discussed in KAMP and KAISER). Under this analysis *Much water ebbs* would be approximately equivalent to *More water ebbs than one normally expects.*

This rather vague formulation has the advantage of pointing to a useful place of development for pragmatic concepts — namely in shaping expectations one places on speakers and their intentions with regard to meaning. It might indicate, for example, why (22) is understood differently in different situations:

(22) Much water is wasted

a) in a conversation about the need for nuclear energy to supplement hydro-electricity. 'much' would be on the order of $10^8$ m$^3$ of water.

b) on a camping trip where the campers have the habit of extinguishing fires with water. 'much' could mean $10^6$ m$^3$ of water.

c) in a debate about a law requiring air-conditioners to recirculate water. 'much' might here be anywhere from $10^3$ to $10^6$ m$^3$ of water, depending on whose law this is to be.

We don't know why expectations should be more specific in connection with adjectives; the following may play a role: adjectives appear with common nouns and thus have more context to shape expectations.

V Plurals

CREASEWELL also claims that his *er than* can handle comparison between plurals such as

(23) More men walk than birds fly

This comes about in the following way. The function of the plural operator *pl* is to create an entity like a mass-noun from a common noun.

Thus, e.g. *<pl, man>* is a two-place predicate satisfied by x, y roughly when x is a y-membered set of men. Since roughly the same predicates which apply to sin-
gular noun phrases apply as well to plural noun phrases, CRESSWELL's regarding the plural as essentially denoting sets of objects has the consequence that predicates will also have to be regarded as applying to sets of objects. But, CRESSWELL would reply, we do this anyway in sentences like

(24) The team walked to Cleveland  

_Men run_ (and _many men run_) is analyzed analogously to _water ebbs_ (and _much water ebbs_). It has the deep structure

(25) \(<\text{deg}, \langle \text{pl, man} \rangle, \text{run} \rangle\)

so will be true in w (according to CRESSWELL's _deg_) iff the number of running men in w is considerably toward the top end of the greater than scale when restricted to the class of men in w. We submit this as further evidence of the need to alter the definition of _deg_ in the direction we've suggested above, since obviously the sentences in (26) require various minima

(26a) Many soldiers march in parades
(26b) Many soldiers mutiny
(26c) Many soldiers are homosexuals

of parading, mutinying or homosexual soldiers to count as true.

To treat numerical expressions like (23), CRESSWELL makes use again of _tot_, which we remind, isolates the element highest on the scale (here: numerical scale). (23) is in fact assigned the following analysis:

(23') \(<\langle \text{tot}, \langle \text{pl, man} \rangle, \text{walk} \rangle, \text{er than}, \langle \text{tot}, \langle \text{pl, bird} \rangle, \text{fly} \rangle \rangle\)

for which CRESSWELL offers the following paraphrase, unobjectionable in content, even if a bit periphrastic:

(23'') The degree of the totality of walking men is greater than the degree of the totality of flying birds.

VI Comparison — A Unified Phenomenon?

We have ignored up till now what might be a serious difficulty in our extension of CRESSWELL's theory of comparison. In postulating the device of mappings to account for mixed comparison, we have opened the door to comparisons of all kinds. The (linguistic, not logical) difficulty lies in the fact that many intuitively nonsensical sentences are assigned non-nonsensical analyses:

(32) * More water ebbs than Bill is tall
     * Ophidia is more beautiful than mud flows

In CRESSWELL's original definition of _er than_ (or rather of \(V(\text{er than})\)) these sentences would be regarded as anomalous, since degrees of different scales are involved. But, if one introduces the possibility of comparison via mappings between
scales, as we have done above, this explanation of the peculiarity of the sentences in (32) evaporates. Several remarks about this problem:

(i) If CRESSWELL’s analysis were correct, we would expect comparison between adjectives and mass nouns to be unproblematic if based on a shared scale. (33) indicates that this isn’t so:

(33)  * This room is more voluminous than water flows

If CRESSWELL’s analysis were correct, we would again require an explanation for the comparisons in

(34)  Ophelia is more beautiful than intelligent

and similar examples (see part III) which, although intelligible, do not involve shared scales.

(ii) (32) is but the tip of an iceberg we can now begin to sound. CRESSWELL’s treatment has the virtue of providing a unified analysis of comparison for adjectives, mass nouns and plurals. The analysis relies on the incompatibility of scales to explain why certain comparatives are not really comparatives, or at least, why they are not easily comprehended. This condition seems too strong, at least for the analysis of adjectives. So we allow for the possibility of comparison between different scales through the device of a mapping from the two onto a third. Here the question is: when is / isn’t such a mapping feasible?

The question is misdirected, however, for it assumes that an answer will be found in terms of mappings. But comparisons are seldom possible as expressed between two different syntactic configurations:

(35)  + Ophelia is taller than there are squares on a chessboard
+ Tom sees more movies than water flows
+ Mary was more heavily burdened than water flows in the Rhine

Although these are semantically plausible:

(36)  ? Ophelia is taller in inches than there are squares on a chessboard
? Tom sees more movies than cubic meters of water flow past the George Washington bridge (per hour)
? Mary was carrying more water than flows in the Rhine

These facts demand a syntactic explanation both in CRESSWELL’s original theory and in the extension we have proposed. The only alternative would be to view the sentences in (35) as well-formed and to invoke some principle regarding the difficulty of processing such sentences. But we have no counterexamples to the syntactic claim and no precise theory of how these sentences are understood. In other words, our extension of CRESSWELL’s treatment incurs no added difficulties in this respect.

The examples show nonetheless that comparison is not grammatically uniform. Even if we retain a unified semantics, some syntactic distinctions are necessary.
(iii) For some purpose, the assignment of too many analyses to a sentence does no great harm. Thus, if we want our semantic theory to provide analyses of natural language argumentation, it is preferable to be too liberal rather than too parsimonious in recognizing "readings" of sentences. For suppose we wish to show that a particular argument in natural language is not valid. Then we need to maintain that under any analysis, the argument is not valid. Then we need to maintain, with some plausibility, that our set of assigned analyses exhausts the possible readings for a sentence. For this purpose it would be preferable to tentatively accept doubtful readings for sentences; this does not contradict the fact that we nonetheless prefer a semantic theory to be exact in assigning analyses to sentences.

As we noted at the end of section III, mixed comparisons can be used in natural language arguments.

VII On the Pragmatics of Comparison

We address here the problem of the extent to which the implications postulated above (for type B adjectives) actually hold.

(37) Tom is (at least) as handsome as a wart-hog.
     Michelle is (at least) as witty as an earth worm.

According to the treatment we proposed, the sentences in (37), if true, imply respectively:

(38) a wart-hog is handsome
     an earth worm is witty

which is, of course, absurd. Then, it might be concluded, the sentences are false. In that case, the sentences in (39) ought to be true.

(39) Tom isn't (even) as handsome as a wart-hog.
     Michelle isn't (even) as witty as an earth worm.

But this is worse. Do the sentences in (37) and (39) presuppose then the sentences in (38)? Our semantics would require only minimal readjustment to deal with this.

We reject this tack, because the problem isn't peculiar to type B adjectives. Strong is a type A adjective; in (40) a. does not imply either clause of b.

(40) a. Tom is stronger than Michelle.
     b. Tom is strong, Michelle is strong.

But now compare:

(41) Tom is (at least) as strong as a wet noodle.
(42) Tom isn't (even) as strong as a wet noodle.

The problem is that (41), (generally) trivially true, has about the same force as (42).
The suspicion presents itself, that the problem has a non-semantic aspect. This is confirmed if we regard the sentences in (37) and (41) in the light of Grice’s conversational maxims, one of which requires: be informative! and, especially relevant, be non-trivial!\(^{26}\)

Suppose, then (contrary to the system proposed above), we regard the sentences in (37) and (41) as having the same basic semantics, i.e. we abolish the distinction between adjectives of type A and adjectives of type B. In these cases, we regard the perceived peculiarity of the sentences in (37) and (41) as simply a result of the maxim admonishing against triviality. In effect, the conversation partner, perceiving that the sentences in (37) and (41) are trivially true, but realizing that the speaker doesn’t wish to be trivial, seeks a ground for these formulations. He might well suppose that the sentences are so formulated because the speaker is acting in accordance with a further norm, viz. to shun explicitly deprecating remarks. The partner concludes then that any true non-trivial remark on this subject would be deprecating. In this way, the sentences in (37) and (39) on the one hand, and sentences (41) and (42) on the other, have the same “force”, they convey very similar information.

But this ‘adjustment’ has some further consequences:

1) as we noted in the text, ‘mixed’ comparisons (i.e. involving adjectives which don’t share a scale) display the following phenomenon: if \(A_1\) is of type A and \(A_2\) of type B, then, in mixed comparisons, the ‘norm’ of \(A_2\) is ‘grafted’ onto \(A_1\).\(^{27}\)

Thus in:

\[
(43) \quad \text{Sam is taller than he is handsome.}
\]

we seem to have the implication that Sam is tall, an implication that is missing in other comparisons involving \textit{tall}. But if we now abandon the type A — type B distinction, how do we account for this? We note i) it probably isn’t an implication in the strict sense. Thus we have no contradiction in (44):

\[
(44) \quad \text{Sam certainly is taller than he is handsome, but he isn’t tall.}
\]

ii) this strong presumption exists in mixed comparisons between adjectives with the same type scales. Compare:

\[
(45) \quad \text{Sam is taller than (he is) strong.}
\]

As we noted in (40), \textit{strong} is of type A. Why then does (45) carry the same strong presumption as (43), that Sam is tall? The suggestion that the norm of type B adjectives is grafted onto the type A scales is (alone) insufficient as an explanation.

2) Before suggesting an explanation for this “strong presumption” that \(A_1\) holds of its subject, let’s consider the “implication” that \(A_2\) holds. In the system we proposed above (43) implies that Sam is handsome, while (45) doesn’t imply that he is strong. But, they may in fact both be “strong presumptions” without being genuine implications. Both (46) and (47) are non-contradictory:

\[
(46) \quad \text{Sam is taller than (he is) handsome. But then he’s not really handsome.}
\]

\[
(47) \quad \text{Sam is taller than (he is) strong. But then he’s not really strong.}
\]
We agree, however, that the presumption is somehow stronger in (43) than in (45), and that (46) is correspondingly somehow less likely than (47). We return to this below.

The problem is similar when the mixed comparisons are present with two subjects, as in (48) and (49):

(48) Sam is taller than John is strong.
(49) She’s as honest as the day is long.

Again there is a strong presumption that John is strong and that the day is long, although no contradiction arises when these presumptions are denied (by some person b):

(48′) ... but John isn’t that strong.
(49′) a:...b: The days in winter, you mean.

But what can all this mean? Is ‘strong presumption’ to be defined model-theoretically? Is it, in fact, a semantic concept?

We suggest that it isn’t. It is preferable to look to the pragmatics of mixed comparison for an explanation of these matters. These sentences seem to appear in very similar contexts. Consider the following dialogue:

(50) a: Shirley is well-read.
     b: She’s more intelligent than (she is) well-read.

b is countering a’s claim, without in fact denying it, even though b may be inclined to deny it. It is for this reason, probably, that informants are unwilling to infer from b’s statement that Shirley is well-read. (50) represents one typical use of mixed comparison, that of countering, without denying, a previous statement. A second typical use is exemplified in (51):

(51) a: I find that Lawrence Durrell is a better writer than Henry Miller.
     After all, he is better-read (than Miller).
     b: But Miller is more intelligent than Durrell is well-read.

In this conversation, there is certainly a presumption that Durrell is well-read. But we would nonetheless refuse to call it an implication of the sentence in b that Durrell is well-read. Rather, it is a partner’s statement which b hasn’t challenged.

In failing to challenge a’s statement, b sanctions it and, in this sense, his utterance “implies” that Durrell is well-read. But this is a very loose, everyday sense of ‘implies’. Compare the following exchange:

(52) a: Durrell is a better writer than Miller. He’s more essential (than Miller).
     b: But Miller is more exciting and intelligent than Durrell is “essential”, if, in fact, he is essential.

In this case, b again hasn’t actually affirmed or denied a’s statement, but has “redoubled”, offering putatively more relevant or more striking superiorities for his
candidate. Unlike b in (51), b here actually calls into question the usual presumption, proving that it's not an implication in the strict sense. (The same thing would be possible in (51) if one could suspend one's knowledge that Durrell is wellread.) A more pragmatic approach would take this tack consistently, ascribing the usual presumptions associated with mixed comparison to the uses they are most often put to. Thus, the mixed comparison involving only one subject (above, as in (43)—(47)), used as it often is to counter another's statements without actually denying them, is essentially free from implication. The presumption that the second adjective holds of the subject stems from the fact that this is often stated before the mixed comparison sentence, and it isn't explicitly denied. Thus the content of the sentence is neutral with respect to the implication. Similarly for the assumption connected with the second half of mixed comparisons involving two subjects, as in (54):

(53) Joe is $A_1$-er than (he is) $A_2$.
(54) Joe is $A_1$-er than Pete is $A_2$.

And the presumption that $A_1$ holds of its subject?

We suggest that this arises essentially via the above discussed presumption and a condition on the mappings between scales we will now discuss. Given (55), how can one conclude (56)? Similarly, how can one conclude from (57) that (58)?

(55) Joe is $A_1$-er than Pete is $A_2$ (stated)
Pete is $A_2$ (presumed)
(56) Joe is $A_1$
(57) Joe is $A_1$-er than $A_2$ (stated)
Joe is $A_2$ (presumed)
(58) Joe is $A_1$

We find no counterexamples to this pattern, but note that it isn't a consequence of our treatment of comparison thus far. We can build the consequence into our treatment by requiring that all mappings are of a certain type, which we will call 'normal'.

A mapping from two scales A and B to a third C is normal iff all elements from the pos-part of each scale are mapped above elements from below the pos-part of the other scale. That is, the following sort of mapping is disallowed:
We recommend, in fact, that this condition be adopted, that in evaluating comparisons, only normal mappings are considered. It will easily be seen that the condition accounts for the validity of (55)–(58). This condition on mappings between scales provides as well a way of analyzing mixed comparisons on a single scale:

\[(5) \quad \text{Bill is taller than Sam is short}^{31}\]

The major remaining problem is to account for, essentially, why one is inclined to assume two different types of scale in the first place. It will be remembered that we originally wished to account for the difference between (59) and (60): In (59) a. does not, in (60) it presumably does entail b.

\[(59) \quad \begin{align*}
  \text{a. Joan is taller than Sam.} \\
  \text{b. Joan is tall, Sam is tall.}
\end{align*}\]

\[(60) \quad \begin{align*}
  \text{a. Joan is more beautiful than Sam.} \\
  \text{b. Joan is beautiful, Sam is beautiful.}
\end{align*}\]

As suggested above (cf. (37)–(42)), we wish to deny that these are implications in the strict sense. While no one will deny this with regard to (59), the situation in (60) is less clear. One is inclined toward the acceptance of the inference. And while the explanation above is sufficient, the possibility isn’t to be dismissed that the sentences in (61) are peculiar because of the failure of this implication to be believable:

\[(61) \quad \begin{align*}
  \text{Joan is more beautiful than a wart-hog.} \\
  \text{Tom is more handsome than a wart-hog.}
\end{align*}\]

We repeat here that we dislike this account of the peculiarity of (61) because it can’t be generalized to cases like:

\[(62) \quad \begin{align*}
  \text{Sam is stronger than a wet noodle.} \\
  \text{Sam is slimmer than some hippopotami.}
\end{align*}\]

We have no neat solution to offer at this point, and can only hope that future research might help to clarify this area. We do note that the tendency to accept the implications here seems to be related to whether the adjective can be sensibly applied to all objects. Thus, for any physical object X, there’s a degree y such that (63) is true:

\[(63) \quad X \text{ is } y\text{-much tall.}\]

But for many adjectives, there’s no such complete metric, even if there’s a metric for some portion of the domain. The adjective *green* is one such example. For many objects, (64) would be meaningless:

\[(64) \quad X \text{ is } y\text{-much green.}\]

Imagine a bright blue and a bright yellow ball; would it mean anything to say that one were *greener* than the other? Clearly it would be difficult to understand if the objects compared weren’t both ‘(somewhat) green’.
While the same conceptual difficulties aren’t as striking with *beautiful* and *handsome*, we perhaps tend to accept the implications here for similar reasons, boiling down finally to: what’s the sense of the comparisons if the sentences supposedly implied don’t hold?

References


Notes

0) The authors are grateful to Thorsten Lorenz for long discussions in preliminary stages of our paper.


2) /SD/, p. 271


5) /SD/, p. 264, 1.9. and /SD/, p. 264

6) This view, i.e. seeing adjectives solely as modifiers, has been challenged by H. KAMP in his article ‘Two Theories about Adjectives’ in Keenan *Formal Semantics of Natural Languages*, 1975, and more recently by G. Kaiser in LB 59, ‘Hoch und gut- Überlegungen zur Semantik polarer Adjektive’.

7) As noted before we dispense with a syntactic distinction between common nouns and intransitive verbs. This of course carries over to their respective modifiers.

8) CRESSWELL does not seem to want to insist on this, cf. /SD/, p. 266.

9) /SD/, p. 268 and pp. 273f.
10) In /SD/, fn. 10 p. 270 CRESSWELL provides one argument for defining \( V \) (\textit{er than}) and \( V \) (\textit{as...as}) as having domains consisting of functions, rather than simply comparing degrees directly. He notes that using his sort of definition we need no extra apparatus to provide the semantics of:

(i) Bill is taller than Arabella or Clarissa

Since his definition effectively requires that the degree of Bill’s tallness exceeds anything that is a degree of Arabella’s tallness or a degree of Clarissa’s tallness. If the definition required comparing two degrees of tallness what would we take as the degree of tallness for “Arabella or Clarissa”? Or would we need a new definition for \textit{er than}? While this argument is technically clever, it falls down at the observation that \textit{and} is as common as \textit{or} in such constructions: (ii) Bill is taller than Arabella and Clarissa.

11) We’re uncertain as to whether (5) is grammatical, but we formulate a semantics in VII that would assign a reading to (5).

12) It is interesting to note that, while most of the mixed examples with \textit{er than} sound unnatural when translated into German, the examples with \textit{as...as} are unproblematic. Compare:

Er sieht besser aus als er intelligent ist

Sie ist genauso attraktiv wie intelligent.


14) These classes could well be built up according to the suggestions G. KAISER (LB 59) makes for multi-dimensional adjectives among which \textit{beautiful} would belong. The final mapping onto a number scale after a number of context-dependent weighing-processes have been carried out would have its counterpart in our assumption that an ordering of these classes on a scale is possible.

15) We may not need the precise notion of ‘average’ at all. CRESSWELL effectively avoids this problem with the locution ‘considerably toward the top of the scale’, see \( V \) (\textit{pos}) below.

16) The definition of \( V \) (\textit{beautiful}) isn’t exhaustive.

17) A function \( f \) is order-preserving iff for all \( a, b \in M \), where \( R_1 \) is an ordering on \( M \), \( R_2 \) an ordering \( f(M) \), then, if \( a R_1 b \), then \( f(a) R_2 f(b) \).

We have to introduce the third scale because one may not be able to find room for all the elements of one scale in another. Nor can we simply attach one scale to the end of the other, since we have to allow for various orderings, in particular (e.g.),

\[ f(u_1) > f(u_2) \quad f(u_2) > f(u_1) \]

where \( u_1, u_2 \in \mathcal{F}(\succ \phi_1) \) and \( u_2 \in \mathcal{F}(\succ \phi_2) \).

18) /SD/, pp. 274–280

19) /SD/, p. 274. In fn. 13 on that page CRESSWELL remarks that this paraphrase is too restrictive since mass-nouns can be measured in several different ways, e.g. weight and volume.

20) This is correct in that both \textit{deg} and \textit{pos} allow us to omit an explicit degree argument. But while \textit{pos} is of category \( \langle 0,1 \rangle \), \( \langle 0,1,1 \rangle \), \( \langle 0,1 \rangle \) \ i.e. makes an ordinary noun modifier out of a modifier such as \textit{tall}, \textit{deg} is of \( \langle 0, \langle 0,1,1 \rangle, \langle 0,1 \rangle \rangle \), that is, it combines a mass-noun and a verb-phrase to form a sentence. Consequently sentences containing \textit{deg} are analysed syntactically in a completely different way. In particular the phrase \textit{much water} in \textit{much water ebbs} is no longer a constituent, i.e. the subject-predicate distinction is relinquished.

21) The definition (/SD/, p. 277) incorrectly restricts the relevant scale to (in this example) \textit{ebbing} water.
22) Here, and below, we would prefer to say 'a lot of' instead of 'much'. There is no change in the semantics.

23) /SD/, p. 277

24) /SD/, p. 278, fn. 17


27) See above, p. 8

28) We note again that this isn't an implication in the strict sense. This is especially obvious in cases where the counterparts to what is presumed in (55) and (57) are explicitly denied:

The public wasn't well-informed about Vietnam.
It was better-informed than it was interested.

29) That is, no relevant counterexamples. Where no mappings are involved in evaluating (55)–(58), counterexamples are easily constructed:

a. Joe has gotten heavy!
b. He's still (a bit) taller than he is wide.

Here we certainly do presume that Joe is wide, but needn't conclude that he's tall. But this is explainable, since tall and wide share the scale of (e.g. metric) length.

30) This is a bit deceptive, in that pos operates on adjectives, not on adjective phrases (i.e. tall instead of tall man). Compare p. 3 above.

But it is always defined relative to the common noun which the adjective applies to, which justifies our phrasing here.

31) See p. 4 above.