Summary

Chapter 1 of this thesis shows that language plays an important role in education. Students are expected to learn from school text books on their own, to listen actively to the instruction of the teacher, to ask questions and to talk to each other about newfound experiences and knowledge. For this reason, students must have sufficient language skills to participate in all subjects (not only language subjects). We know that students with low language abilities experience problems in many subjects. Not all students seem to have sufficient language proficiency allowing them to cope with school demands. It is estimated that at the start of secondary education, 20 to 30% of students are not able to comprehend school text books. Recent research shows that many students leave school while still having problems with written tasks.

Mathematics is one of the subjects in secondary education for which students' language skills have become more important. In the seventies, mathematics education in the Netherlands underwent a number of serious changes. Since the implementation of Realistic Mathematics Education (RME) developed by Freudenthal, language and text comprehension have become more important in the teaching of mathematics. According to this didactic theory, students must themselves (re)invent mathematical ideas. To stimulate this (re)invention of mathematics, the starting point of instructional experiences in RME should be 'real' to the students, allowing them to immediately engage in the situation. This means that all mathematical problems are placed in contextual problems right at the start of the learning process. These contextual problems must be described by using language. Therefore, the amount of language increased substantially in RME mathematical textbooks compared to traditional mathematical textbooks. Moreover, according to the RME view on mathematics education, students should be active learners. To participate as active learners of mathematics, students must talk and listen to each other, showing once again the importance of students' language proficiency. For students who are less proficient, oftentimes non-native students, the increasing use of language in mathematics education could be an obstacle. In the research described in this thesis, we examined to what extent language skills, particularly text comprehension, are important in mathematics education and we investigated the individual processing of math problems by students.

In *chapter 2* we determine that the first important step in solving a mathematical problem consists of reconstructing the text of that mathematical problem. Students must understand the text of the problem before they can solve it. To describe the first step in the process of problem solving, we adopted Van Dijk and Kintsch's interactive reading model. Their model of text comprehension describes the complete reading process, from recognizing words to constructing a representation of the meaning of the text. According to this model, the reader starts by analysing the text to make *a surface representation* of the text. Based on a list of propositions derived from the text, the reader starts to construct a *propositional representation* (the text base). By using situational knowledge, this list of propositions is transformed into a network of coherent propositions in the *situation model*. Following Hacquebord, who bases her research on Van Dijk and Kintsch's model, we distinguish three levels of text processing: the micro level, the meso level and the macro level. The micro level is the level where the analysis of words in a text takes place. The meso level is

the level where connections within and between sentences are made, and the macro level is the level of analysis for the complete text, the overall text structure.

The importance of several aspects of text comprehension in school textbooks and mathematical textbooks is investigated on the basis of this classification of micro, meso and macro levels. We developed several tests to review the skills of students at these levels. To examine the skill of *text comprehension of school textbooks*, we used a test for school words (micro level), a test for determining the skill of comprehending the paragraph (meso level) and a test for general text comprehension of school text books (all levels). For determining the skill of *text comprehension of mathematical texts*, we used a test for math words (micro level) and a test for general text comprehension of mathematical texts (all levels).

First of all, we looked at the differences in performance on different tests between non-native and native students. We observed that for the examined population of vocational students in their second term at school, non-native students performed poorer on text comprehension of mathematical texts, the mesoniveau of text comprehension, the knowledge of school words and on the knowledge of math words. For the text comprehension scores on general school textbooks, we found no difference in performance.

The results show that proficiency in *text comprehension of general school textbooks* is commonly predicted by performance on the meso level and by knowledge of school words. These relationships are different for non-native and native students. For native students, only the skill on the meso level is a predictor of text comprehension of general school textbooks, whereas for non-native students only the skill at the micro level, the knowledge of the school words, is a predictor of text comprehension of general school textbooks.

Subsequently, if we examine the proficiency of *text comprehension of mathematical texts* in particular, then we see that performance on this skill for the complete group of vocational students is predicted by performance on the skill of text comprehension of general school textbooks, skill on the meso level and knowledge of school words. If we distinguish between non-native and native students, we see that for the individual groups only the score on general text comprehension and the score on school words predict text comprehension of mathematical texts. The score on the meso level however does not seem to predict text comprehension of mathematical texts.

In conclusion, we can state that both for native and for non-native students, text comprehension of general school textbooks and text comprehension of mathematical texts in particular, are predicted by knowledge of school words. Because non-native students score lower on knowledge of these school words than native students, we can cautiously say that the lower language proficiency of these students impedes their understanding of mathematical texts. However, non-native students do not score lower on general text comprehension than native students. These students are most likely able to compensate their language proficiency problems on the micro level of the text with skills on the macro level in comprehending general school textbooks, but are unable to compensate these problems on the micro level in comprehending mathematical texts.

In *chapter 3* we examined the linguistic features of a chapter from a mathematical textbook. We looked for potential obstacles for students on both the textual microniveau of the math texts (word level) and the textual meso level of the math text (paragraph level). We made an inventory of the complexity of the text characteristics of texts from a chapter from a math book on graphs. For the textual micro level, we did this by calculating the degree of coverage in mathematical texts using two external criteria: the so-called *Basic Words List* ('Basiswoordenlijst') and the so-called *Target Word List for the first years of secondary education* ('Streefwoordenlijst voor de Basisvorming'). To analyse the complexity of the textual meso level, we asked a panel to review the texts.

The analyses at the textual micro level show that the math book contains many infrequent everyday words, such as 'hartslag' (heartbeat), 'populier' (poplar) and 'brandglas' (burning-glass). We have seen that unfamiliar words such as these do not have to be an obstacle for students in each mathematical problem. Only when it is necessary for students to understand the context of the problem, is it possible that an unknown word from daily life can impede students' understanding of the problem. Students with limited word knowledge, for example non-native students, will probably have more problems with unknown everyday words. Furthermore, the results of the analyses show that new, important, mathematical terms are introduced in a very contextual way. As a consequence, these terms are not clearly defined. This could also be an obstacle for students with low language abilities.

On the textual meso level, the panel noted the short and concise manner in which text is formulated. The panel determined that this manner of formulating texts does not always contribute to increased text comprehension. Generally speaking, we get the impression that authors are inclined to limit the amount of verbal information given to students - most likely in an attempt to contain the quantity of language in math books. Moreover, it was noted that different terms were used to refer to the same referent on multiple occasions. Very few explicit clues were used at the meso level to mark relations in the text. This can lead to obstacles for students with low language abilities.

In *chapter 4* we looked at how individual students process one specific mathematical problem on graphs. First, we analysed a math problem from this field and then we asked students to solve the problem while thinking aloud.

For the analysis of the problem and the analysis of the thinking aloud protocols, we used English and Halford's adapted model for problem solving. This model is based on the ideas of Van Dijk and Kintsch. In this model, the process of solving a mathematical problem concerning graphs is described. We stated that there are two sources of information for the reader in such a mathematical problem: the textual information (the text of the problem and the text accompanying the graph) and the visual information (the graph itself). A reader solving a mathematical problem first constructs a *problem-text model* by means of a propositional analysis of the text. On the basis of this problem-text model, the reader uses knowledge of the world and the situation to come to the construction of a *problem-situation model*. Moreover, on the basis of the graph (the visual information) the reader can construct a *graphic model*, in which the information can be used that is visible in the graph. The graph becomes meaningful if the reader incorporates the information from the *problem-situation model* and the *graphic model* into the *mathematical model*.

We then asked students to solve the mathematical problem while thinking aloud. We analysed the protocols of these thinking aloud sessions based on demonstrations of comprehension and incomprehension. The results showed that there are students who demonstrate comprehension of the mathematical problem and students for whom it is incomprehensible. Several indicators in students' language use allowed us to say whether or not students correctly constructed the different interpretation models. Incomprehension appeared to be caused by (1) problems with the reconstruction of the text (the problem-text model), (2) problems with the graph (the graphic model) or (3) problems with interpreting the situation (the problem situation model), sometimes in combination with the graph (the graphic model, incorporated in the mathematical model). Moreover, students could even demonstrate comprehension while (4) having difficulties formulating the answer correctly.

While solving the problem, relatively many students (more than half) appeared to have problems reconstructing the text. We can identify several gradations of incomprehension within these problems en we can conclude that many students stumble on the linguistic context of the problem. Particularly on the micro level, students evidently experienced problems.

In *chapter 5,* we looked at the language use of students when formulating an answer to a question in the mathematical problem as well as their language use in explaining their answer. We examined these questions within the same context of the mathematical problem concerning graphs. We only looked at the questions in which students had to characterise the curve in the graph: the (rapid) increase of the curve and the decrease of the curve.

The results for the two concepts (increase and decrease) differed. While describing the more rapid increase of the curve, students used mainly everyday terms or a combination of daily and mathematical language to describe the characteristic of the graph. However, students mainly used mathematical terms to describe the decrease of the graph. We also saw a shift in students' language use in explaining their answer: most students tried to paraphrase their answer using more formal language.

We observed that some students are successful in solving a mathematical problem without using mathematical language. They know the mathematical concepts necessary to solve the problem, but do not use mathematical terms to refer to these concepts. They use daily language to solve math problems.

Yet, according to the foremost goals of mathematics education, students must learn to use appropriate mathematical language. Students can experience difficulties if they cannot express themselves appropriately. We saw this with students who had problems describing the characteristics of the graph. They used a mathematical or everyday term not suitable for explaining the difference in increase of the curve. Because of this, their first answer was incorrect, despite demonstrating comprehension of the problem.

In this thesis we showed that text comprehension and language skills are important in mathematics education. Since the implementation of Realistic Mathematics Education all mathematical problems have been offered in contexts to stimulate students to use their own strategies and insights in problem solving. Therefore, mathematics education has become more linguistic: the mathematical text book contains more language and students should talk more to each other to create knowledge. This new kind of education offers possibilities for integrated language and content instruction: students are able to understand mathematics better and at the same time they get the opportunity to increase their language skills. However, in practice it appears that these chances for integrated language and content instruction are not always exploited. In this thesis it becomes clear that Realistic Mathematics Education not only offers chances, but also can cause problems for students with low language abilities.

As a result of future didactic changes in the first years of secondary education in the Netherlands, more opportunities for interaction and contextual support will grow. It is important not to forget about the special care that students with low language abilities need. By focusing on the linguistic aspects of mathematics education and paying attention to the language skills needed by students to participate in secondary education, we can ensure that more students have language skills sufficient enough to actively participate in mathematics education as well as a variety of social situations where mathematics play a role.