

# LANGUAGE AND INFERENCE

---

**Day 1: Types of Inference**

**Day 2: Designing Meaning Representations**

**Day 3: Building Meaning Representations**

**Day 4: Projection and Presupposition**

**Day 5: Inference in the Real World**



university of  
groningen

johan.bos@rug.nl

# Language

## Natural Language

Spoken by humans

- English, Dutch, ...
- Italian, German, ...

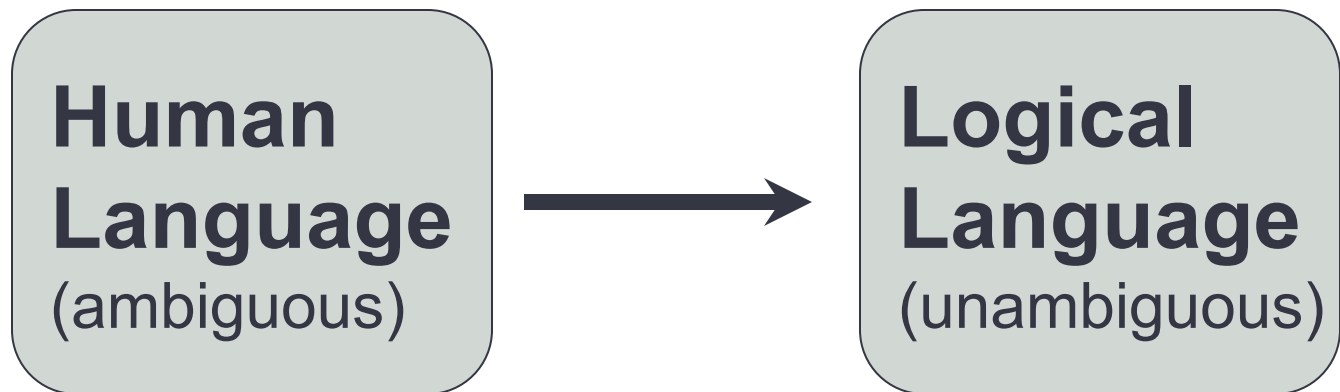
## Artificial/Formal Language

“Spoken” by machines

- Logical languages (calculi)
- Programming languages

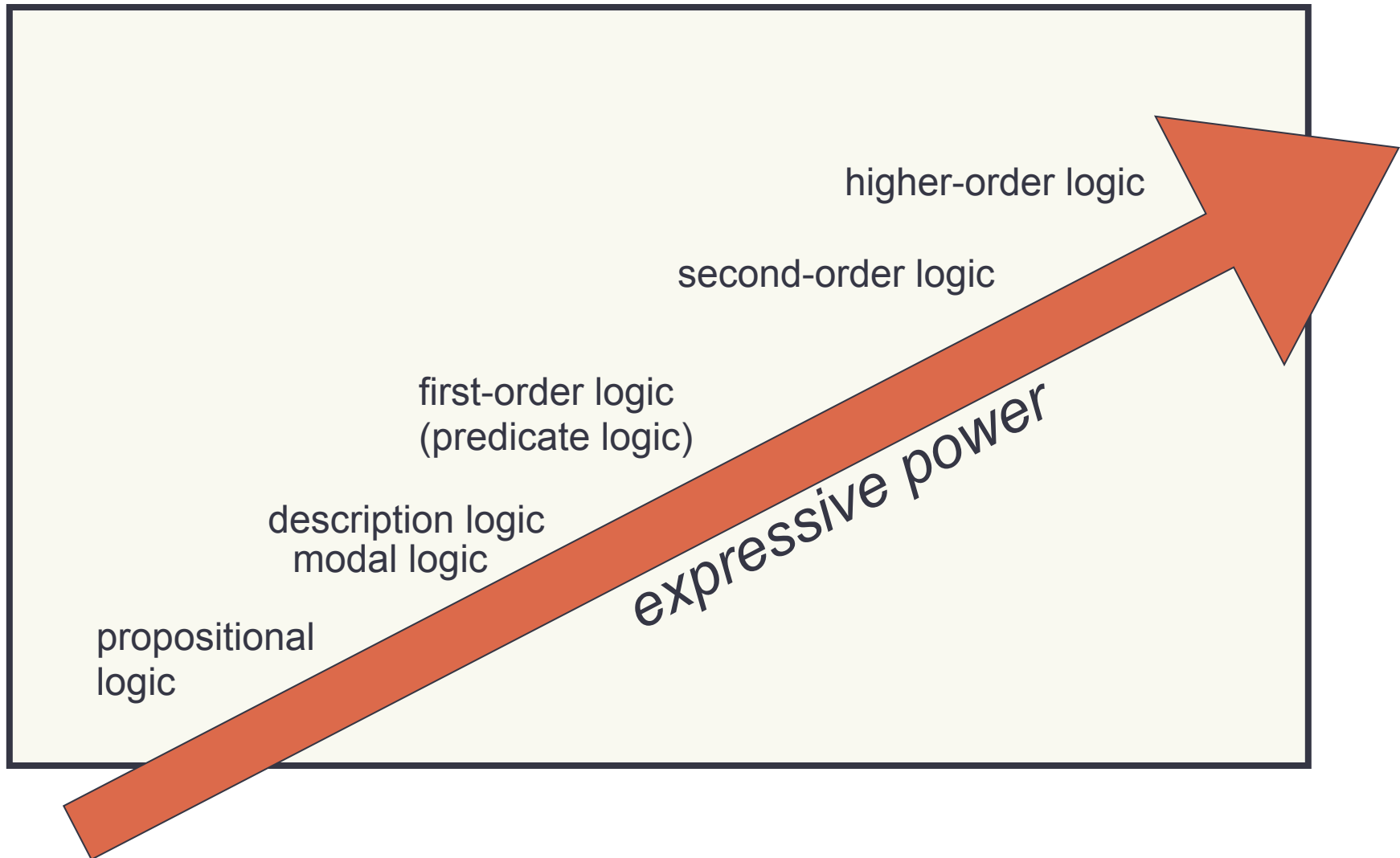
# Basic idea of formal semantics

- Provide a mapping from ordinary language to logic
- Aim: predict inferences (study of meaning)



But what are logical languages or calculi ?

# Logical languages



# Inference

- “Making explicit what is implicit”
- “Drawing conclusions from premises”
- “Gaining knowledge through reasoning”

# Types of inference

- Abductive reasoning
- Inductive reasoning
- Deductive reasoning

# Inference games

**Can we derive the conclusion from the premises?:**

Every boy smiled.	}	<i>premises</i>
Mia is a woman.		
<hr/>		
Mia smiled.	}	<i>conclusion</i>

Note:

- Can be read as: “if” premise “and” premise “then” conclusion
- References (e.g. names) and contexts are considered constant

# Abductive reasoning (Abduction)

**Guessing for an explanation...**

The dog is wet.

---

? It's raining outside.

? It jumped in the pool.



# Inductive reasoning (Induction)

## Making generalizations...

This dog has four legs.

That dog has four legs.

And that one. And this one.

And that one too.

---

All dogs have four legs.

# Inductive reasoning (Induction)

## Making generalizations...

This dog has four legs.

That dog has four legs.

And that one. And this one.

And that one too.

---

All dogs have four legs.

# Deductive reasoning (Deduction)

## **Drawing conclusions from a set of premises**

Every dog jumped in the pool.

Fido is a dog.

---

Fido jumped in the pool.

# The Inference Tasks

- Concentrate on **deductive** reasoning
- With the help of (first-order) **logic**
- Two inference tasks:
  - ① **informativeness/entailment** checking
  - ② **consistency/contradiction** checking

# Single sentence games

For each example sentence: judge whether it is:

- consistent (true in at least one situation) or
- inconsistent (false in every situation)
  
- informative (false in at least one situation) or
- uninformative (true in every situation)

# Example 1: Bush

“... when there's more trade, there's more commerce.”

**informative?  
consistent?**

George W. Bush, at the Summit of the Americas in Quebec City, April 21, 2001 (source: Language Log 24/10/2004)

## Example 2: Snowden

“Information chiefs in many countries sound alarm over revelations by Edward Snowden”

The Guardian 11/06/2013

**informative?**  
**consistent?**

## Example 3: Venice

Turn (simultaneously) left and right to go to San Marco.

**informative?**  
**consistent?**



# Multi-sentence games

For each example: judge whether a new contribution is:

- consistent (possible to be true in a situation) or
- inconsistent (impossible to be true in a situation) wrt previous text
  
- informative (possibly true and false) or
- uninformative (true in any situation) wrt previous text

## Example 1 (a):

The king had cornflakes for breakfast this morning.

---

The king had cornflakes for breakfast.

**informative?**  
**consistent?**

## Example 1 (b):

The king had cornflakes for breakfast this morning.

---

The queen had cornflakes for breakfast.

**informative?  
consistent?**

## Example 2 (a):

Bob and Sue are married.

---

Bob and Sue are married to each other.

**informative?  
consistent?**

## Example 2 (b):

Bob is married to Sue.

---

Sue is married to Bob.

**informative?  
consistent?**

## Example 3 (a):

Jules eats a kahuna burger every day.

---

Jules eats a kahuna burger every Tuesday.

**informative?**  
**consistent?**

## Example 3 (b):

Jules eats a kahuna burger every Tuesday.

Jules eats a kahuna burger every day.

**informative?  
consistent?**

## Example 3 (c):

Jules eats a kahuna burger every day.

---

Jules eats a big kahuna burger every day.

**informative?**  
**consistent?**



## Example 3 (d):

Jules eats a big kahuna burger every day.

Jules eats a kahuna burger every day.

**informative?**  
**consistent?**

# Determiners summed up

- **every**(↓,↑)  
every boy runs -> every small boy runs  
every boy runs quickly -> every boy runs
- **a**(↑,↑)  
a small boy runs -> a boy runs  
a small boy runs quickly -> a small boy runs
- **no**(↓,↓)  
no boy runs -> no small boy runs  
no boy runs -> no boy runs quickly

## Example 4 (a):

Jerry is a large mouse.

Every mouse is an animal.

---

Jerry is an animal.

**informative?**  
**consistent?**

## Example 4 (b):

Jerry is a large mouse.

Every mouse is an animal.

---

Jerry is a large animal.

**informative?**  
**consistent?**

## Example 4 (c):

Jerry is a brown mouse.

Every mouse is an animal.

---

Jerry is a brown animal.

**informative?**  
**consistent?**

## Example 5 (a):

Marsellus is a clever person.

---

Marsellus is clever.

**informative?  
consistent?**

## Example 5 (b):

Marsellus is a clever criminal.

---

Marsellus is clever.

**informative?  
consistent?**

## Example 6

Bolt is faster than Powel.

---

Powel is faster than Bolt.

**informative?  
consistent?**

**informative?  
consistent?**



# Example 7

No pets are allowed in this area.

---

All pets must be on the leash in this area.

**informative?**  
**consistent?**

## Example 8

Steve visited only Bologna.

---

Steve visited Bologna and Pisa.

**informative?**  
**consistent?**

## Example 9 (a):

We bought fresh milk last week.

Today we drink what we bought last week.

---

Today we drink fresh milk.

**informative?**  
**consistent?**

## Example 9 (b):

We bought fresh milk last week.

So did the neighbours.

---

The neighbours bought fresh milk last week.

**informative?**  
**consistent?**

# Example 10

Bill ordered a beer.

John ordered one too.

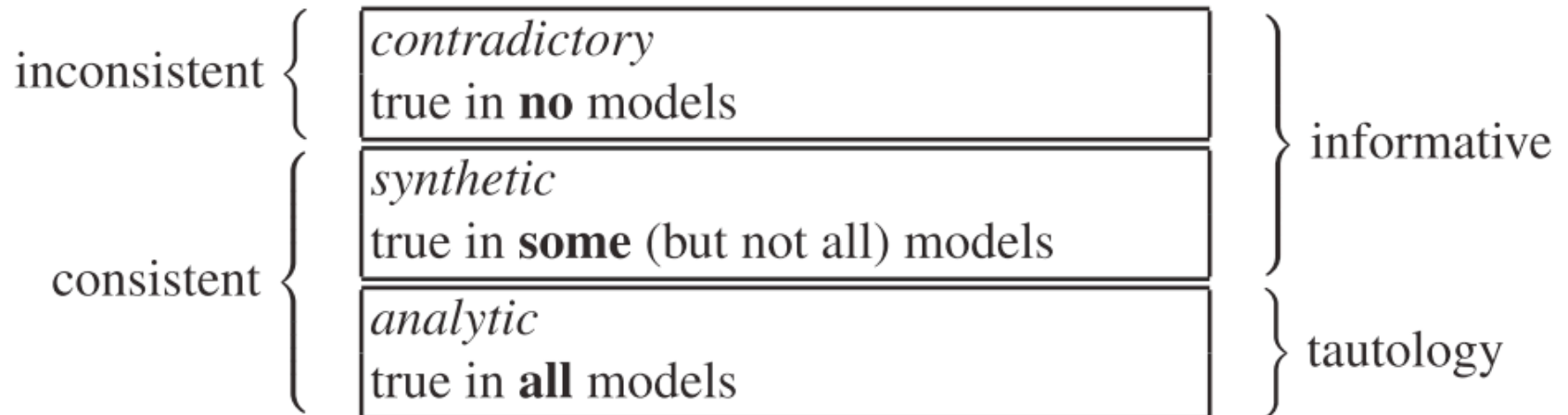
---

John ordered a beer.

**informative?**  
**consistent?**

# Terminology

(Matthews 1997, Bos 2011)



# Grice's Maxims

- Maxim of quality
  - do not say what you believe to be false
  - do not say that for which you lack evidence
- Maxim of quantity
  - Make your contribution as informative as is needed
  - Do not make your contribution more informative than needed

# Entailments & Paraphrases

- Observation 1:  
**Text T entails sentence S iff S is not informative wrt T (i.e. contains no new information)**
- Observation 2:  
**Two sentences are paraphrases of each other iff they entail each other**



# Semantic relations

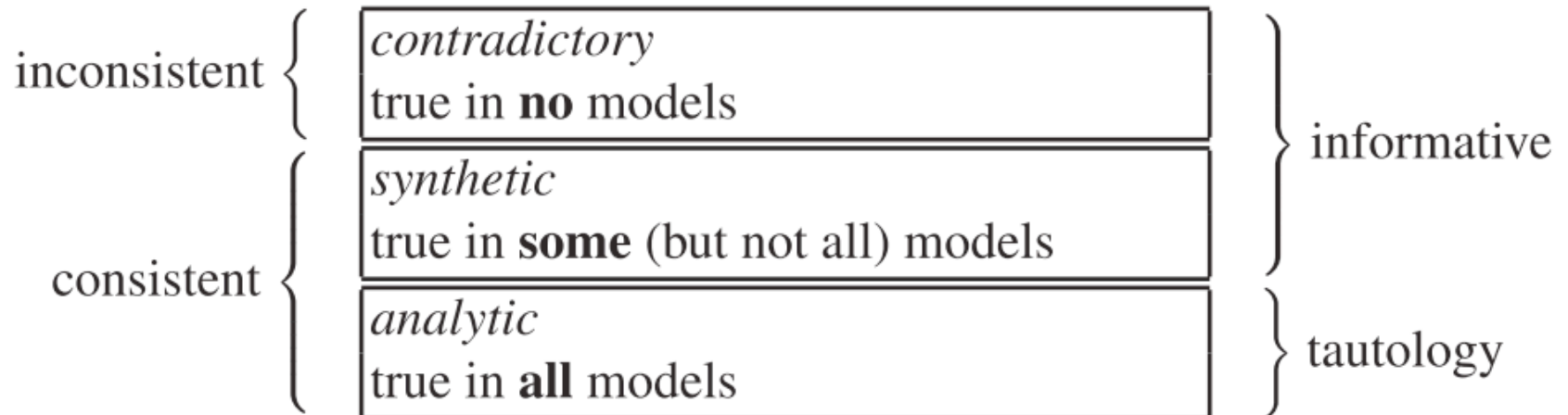
<b>Relation between sentences</b>	<b>Relation between words</b>
entailment	...
paraphrase	...
contradiction	...

# Semantic relations

<b>Relation between sentences</b>	<b>Relation between words</b>
entailment	hyponymy
paraphrase	synonymy
contradiction	antonymy

# Terminology

(Matthews 1997, Bos 2011)



# Models

- Model-theoretical semantics
- Alfred Tarski

Models: approximations of reality

Models: approximations of reality

# Interpretation

- A model *satisfies* a sentence
- A sentence  $S$  is true in a model  $M$   
( $M$  satisfies  $S$ ,  $M$  is an interpretation of  $S$ )

$M$ :

$M$  satisfies “The dog is outside”

$M$  does not satisfy: “A bird sits on a car”

# An example model

---

$M = \langle D, F \rangle$

$D = \{d1, d2, d3, d4, d5, d6, d7, d8\}$

$F(\text{man}) = \{d1\}$

$F(\text{woman}) = \{d2\}$

$F(\text{house}) = \{d3, d4\}$

$F(\text{dog}) = \{d5\}$

$F(\text{bird}) = \{d6\}$

$F(\text{tree}) = \{d7\}$

$F(\text{car}) = \{d8\}$

$F(\text{happy}) = \{d1, d2\}$

$F(\text{near}) = \{(d5, d2), (d2, d5)\}$

$F(\text{at}) = \{(d6, d3)\}$



# Vocabularies (Signatures)

- Ensuring that descriptions and situations belong together
- Example vocabulary:

{ love:2, hate:2, man:1, woman:1, mia:0, vincent:0) }

This tells us: (a) the topic of conversation  
(b) the language of the conversation

# An example model

$M = \langle D, F \rangle$

$D = \{d1, d2, d3, d4\}$

$F(\text{mia}) = d1$

$F(\text{honey-bunny}) = d2$

$F(\text{vincent}) = d3$

$F(\text{yolanda}) = d4$

$F(\text{customer}) = \{d1, d3\}$

$F(\text{robber}) = \{d2, d4\}$

$F(\text{love}) = \{(d4, d2), (d3, d1)\}$

# An example model

---

$M = \langle D, F \rangle$

$D = \{d1, d2, d3, d4\}$

$F(\text{mia}) = d2$

$F(\text{honey-bunny}) = d1$

$F(\text{vincent}) = d4$

$F(\text{yolanda}) = d3$

$F(\text{customer}) = \{d1, d2, d4\}$

$F(\text{robber}) = \{d3\}$

$F(\text{love}) = \{\}$

# A very small model

---

$M = \langle D, F \rangle$

$D = \{d5\}$

# A very large model

---

$M = \langle D, F \rangle$

$D = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}\}$

$F(\text{man}) = \{d_1, d_4, d_{12}\}$

$F(\text{woman}) = \{d_2, d_3\}$

$F(\text{car}) = \{d_{14}, d_{13}\}$

$F(\text{love}) = \{(d_2, d_1), (d_4, d_4)\}$

$F(\text{hate}) = \{(d_5, d_1), (d_1, d_4), (d_2, d_2)\}$

$F(\text{chopper}) = \{d_{10}\}$

# Ingredients of a first-order language

1. All **symbols** in the vocabulary – the non-logical symbols of the languages
2. Enough **variables** (a countably infinite collection):  
x, y, z, etc.
3. The boolean **connectives**  $\neg$  (negation),  $\wedge$  (conjunction),  $\vee$  (disjunction), and  $\rightarrow$  (implication)
4. The **quantifiers**  $\forall$  (the universal quantifier) and  $\exists$  (the existential quantifier)
5. Some **punctuation** symbols:  
round brackets and  
the comma.



# George Boole

pioneer of modern  
mathematical logic

# First-order terms

- Any constant or any variable is a first-order term
- Terms are the noun phrases of first-order languages
  - constants are first-order analogs of proper names
  - variables are first-order analogs of pronouns



# Atomic formulas

- If  $R$  is a relation symbol of arity  $n$ , and  $t_1, \dots, t_n$  are terms, then  $R(t_1, \dots, t_n)$  is an atomic formula
- If  $t_1$  and  $t_2$  are terms, then  $t_1 = t_2$  is an atomic formula

# Well formed formulas (wffs)

1. All atomic formulas are wffs
2. If  $\varphi$  and  $\psi$  are wffs,  
then so are  $\neg\varphi$ ,  $(\varphi \wedge \psi)$ ,  $(\varphi \vee \psi)$ ,  $(\varphi \rightarrow \psi)$
3. If  $\varphi$  is a wff, and  $x$  is a variable,  
then both  $\exists x\varphi$  and  $\forall x\varphi$  are wffs
4. Nothing else is a wff

# Free and Bound Variables

- A variable is free in a formula  $\psi$  if it is not bound in  $\psi$
- A variable  $x$  is bound in a formula  $\psi$  if it appears in the scope of a quantifier  $\exists x$  or  $\forall x$  in  $\psi$

# Closed formulas

- Formulas that have no free variables are called *closed*
- Usually we're only interested in closed formulas
- Translating a natural language sentence to first-order logic should produce a closed formula
- Free variables can be thought of as “pronouns”

# Logicians are only human

- Logicians (and mathematicians) are usually very precise in their formulations
- However, they sometimes drop punctuation symbols if no confusion arises
- Often outermost brackets are dropped; also other brackets as long as no confusion arises
- Examples:
  - $p \wedge q$  instead of  $(p \wedge q)$
  - $p \vee (q \wedge r)$  instead of  $(p \vee (q \wedge r))$
  - $(p \vee q \vee r)$  instead of  $(p \vee (q \vee r))$

# A note on notation...

- Negation:  $\neg$  or  $\sim$
- Conjunction:  $\wedge$  or  $\&$
- Implication:  $\rightarrow$  or  $\supset$
- Equivalence:  $\leftrightarrow$  or  $\equiv$
- Brackets:  $(\dots)$  or  $[\dots]$

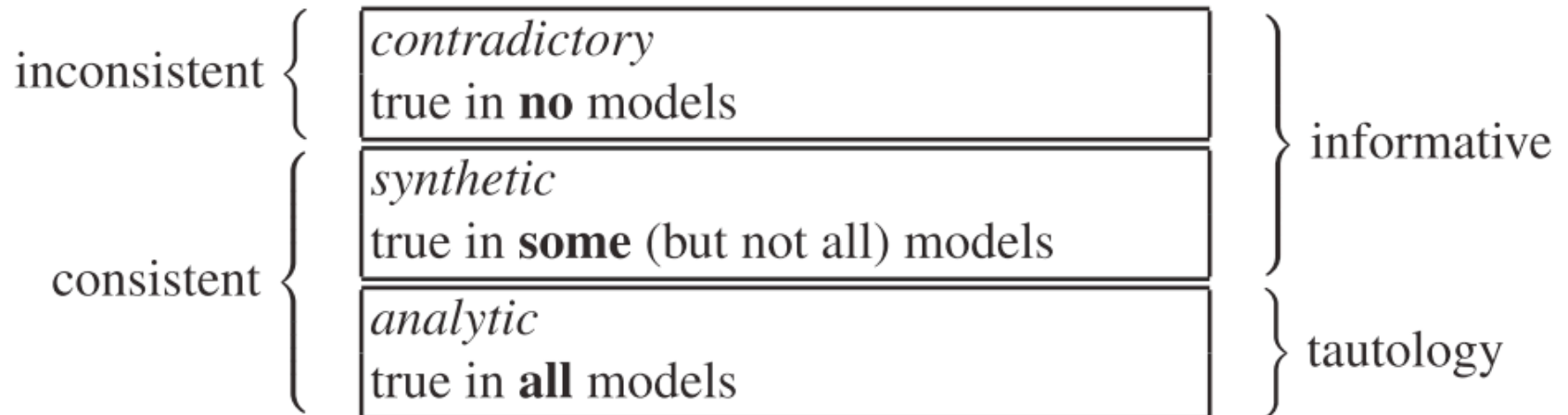
# The satisfaction definition

$M, g \models R(\tau_1, \dots, \tau_n)$	<i>iff</i>	$(I_F^g(\tau_1), \dots, I_F^g(\tau_n)) \in F(R),$
$M, g \models \tau_1 = \tau_2$	<i>iff</i>	$I_F^g(\tau_1) = I_F^g(\tau_2),$
$M, g \models \neg\phi$	<i>iff</i>	not $M, g \models \phi,$
$M, g \models (\phi \wedge \psi)$	<i>iff</i>	$M, g \models \phi$ and $M, g \models \psi,$
$M, g \models (\phi \vee \psi)$	<i>iff</i>	$M, g \models \phi$ or $M, g \models \psi,$
$M, g \models (\phi \rightarrow \psi)$	<i>iff</i>	not $M, g \models \phi$ or $M, g \models \psi,$
$M, g \models \exists x\phi$	<i>iff</i>	$M, g' \models \phi,$ for some $x$ -variant $g'$ of $g,$
$M, g \models \forall x\phi$	<i>iff</i>	$M, g' \models \phi,$ for all $x$ -variants $g'$ of $g.$

$I_F^g(\tau)$  is  $F(c)$  if the term  $\tau$  is a constant  $c$ , and  $g(x)$  if  $\tau$  is a variable  $x$ .

# Terminology

(Matthews 1997, Bos 2011)





# Do we really need all this stuff?

- implication
- disjunction
- quantifiers

## What's wrong with these translations?

English	First-order logic
A dog barks.	$\exists x(\text{dog}(x) \rightarrow \text{bark}(x))$
Vincent likes every dog.	$\forall x(\text{dog}(x) \wedge \text{like}(v,x))$
No dog barks.	$\exists x(\text{dog}(x) \wedge \neg \text{bark}(x))$
Every dog chases a cat.	$\forall x(\text{dog}(x) \rightarrow \exists y(\text{cat}(y) \wedge \text{chase}(y,x))$