Approaching quantitative accuracy in early Dutch city maps

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Introduction

We are motivated to assess the accuracy of early maps both for its own value and also as a step that is needed if one is to integrate the information from various maps in a way that takes differences in accuracy into account. In order to measure accuracy in early maps we have made use of U.S. *National Standard for Spatial Data Accuracy* (NSSDA). These standards were developed to measure and report geographic data quality and are supplemented with a procedure described in the *Positional Accuracy Handbook (1999).* Positional horizontal accuracy was measured in a set of six early maps of the city of Zwolle and the results compared at an aggregate level in order to assess the global accuracy of the maps, and also to compare specific categories and subcategories (classes). This is done in order to understand the variation of accuracy in the maps, more specifically among their features. This information in turn was used to conjecture about the purpose of the maps, under the assumption that important elements are represented more accurately.

A modern digital map (2004) and the cadastral map of 1832[1] of the city of Zwolle and surroundings were used as geographical references. By following the steps for measuring NSSDA standards we have measured positional errors, RMSE errors and NSSDA horizontal accuracy in old maps. Results are analyzed and compared in this document.

NSSDA and positional accuracy

Map accuracy standards are specifications of accuracy requirements on maps.[2] These standards have different specifications for horizontal accuracy (horizontal coordinates x,y)

[1] The modern map 2004 was used as highly accurate source for geo-referencing the cadastral map 1832, which was used as a base to geo-reference the set of six old maps of Zwolle. The cadastral map 1832 was used as an intermediate source due its similarity to the old maps (absent in the modern source). The cadastral map 1832 links modern and old maps, representing also a highly accurate reference source.
[2] U.S. Department of the Interior, U.S. Geological Survey, Maintainer: National Mapping Program Standards Team. Consulted on 01-dec-2006. URL: http://rockyweb.cr.usgs.gov/nmpstds/nmas647.html. The most recent information available on the topic of Accuracy Standards can be found on the Federal Geographic

and vertical accuracy (elevation or depth). NSSDA is the horizontal accuracy statistic tested at a 95% confidence level. By measuring this statistic we tested how accurate old maps are according to the standards of modern cartography.

Geoprocessing of early maps

In order to assess accuracy in early maps we need to be able to compare historical and modern sources spatially by giving a common 'real-world' geographic reference to all maps. We used the GBNK[3] modern map of Zwolle (2004) as a geographical reference as a modern reference system for the cadastral map of 1832, which was in turn used as more highly accurate map for geo-referencing the early maps. [4]

Three different transformations were used for geo-referencing old maps and for analysing differences in results. These transformation classes restrict the sorts of distortions allowed in aligning one map with another. We shall not concern ourselves with the specific assumptions of the different transformations, but we examine three different sorts in order to verify that our work is not dependent on a single sort. We experiment therefore with Two point (4 parameters), Affine (6 par.) and Helmert (4 par.) transformations, and for each of these we calculated the parameters necessary to align the position, orientation and size of old maps with the cadastral map 1832. In the first transformation (two-points) only two reference points, strategically located[5] in the external part of two bastions, were used. In the other two transformations several reference points were used.

Once all maps were aligned in a common reference system, coordinate values (x,y) were collected for a set of common points. These were used for the measurement of positional errors and the assessment of horizontal accuracy.

Data Committee (FGDC) web site (http://www.fgdc.gov), including contributions of the American Society for Photogrammetry and Remote Sensing (ASPRS, http://www.asprs.org/resources/standards.html) as Accuracy Standards for Large-Scale Maps.

[3] GBKN (*De grootschalige basiskaart Nederland*) of Zwolle (2004). Gemeente Archief Zwolle.

[4] Because of the gap in time and content between modern and early maps, the cadastral map 1832 served as intermediate source facilitating the interpretation of features and the taking of measurements from map to map.

[5] Several tests using different pairs of diagonally opposing bastions where made in order to find the best fitting. In these tests the maps were scaled, rotated and translated by using different bastions baselines. From these tests it was concluded that the best fitting between the cadastral map and the old maps is obtained by using the longest distance between bastions However, since one of this points is not represented in the cadastral map, it is correct to note that we may have introduced an initial error in the location of this point.

Positional error

Positional error is given by the difference in the position of a point identified in the early map with respect to the position of the same point in the cadastral map of 1832. This error (magnitude and direction) was measured for every point of the set. The linear displacement (magnitude) of every point is calculated by measuring the Euclidean distance between the two points based on the coordinates x and y. This distance represents how far the location of the point in the early map is from the "real" location[6]. The direction is calculated by the Euclidean angle of the vectors separating homologous points and defines the direction in which the error occurs.[7]

In this paper we focus on the analysis of NSSDA horizontal accuracy therefore positional errors are not analysed in detail.

RMSE and horizontal NSSDA accuracy

We apply the steps described in Positional Accuracy handbook[8] to calculate and report NSSDA horizontal accuracy in a set of early maps of the city of Zwolle. This includes a selection of tests points, selection of the cadastral map 1832 as independent data set of higher accuracy and the respective measurements, calculations and report.

Using these points, the positional accuracy was computed by calculating three values: the sum of the squared differences between the early maps' coordinate values and the coordinate values of the cadastral map of 1832; the mean obtained by dividing the sum of squares by the number of test points being evaluated, and the root mean squared error (RMSE)[9] statistic, which is simply the square root of the mean. The NSSDA statistic is determining by multiplying the RMSE by a value that represents the standard error of the mean at the 95% confidence level (NSSDA Positional accuracy handbook) as explained in

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[6] The location of the points in the geo-referenced cadastral map 1832 is assumed as the "real" location.
[7] To calculate these values we make use of the function arctangent, or inverse tangent, of the differences in x- and y-coordinates between the early maps and the cadastral map 1832.
[8] Minnesota Planning Land Management Information Center, 1999
[9]RMSE error is "the square root of the average of the set of squared differences between dataset coordinates values from an independent source of higher accuracy for identical points ". FGDC-STD-007.3-1998, p3-4

the document of Geospatial Positioning Accuracy Standards FGDC-STD-007.3-1998 in the Appendix 3-a (normative). This procedure assumes that systematic errors has been removed so that error is normally distributed and independent in the x- and y-dimensions. We estimate the number of standard errors needed for a 95% confidence interval at 2.4477 based on the *t*-distribution with 19 degrees of freedom, which we did not vary even where data were sparse in order to have comparable values. Then horizontal accuracy at the 95% level may be calculated by using a formula such as the following. When $RMSE_x = RMSE_y$ (=RMSE), we exploint the fact that RMSE = $sqrt(2*RMSE_x^2)$ = 1.4142 $RMSE_x$, so that $RMSE_x$ = RMSE/1.4142 to use the formula:

$$Accuracy_r = 2.4477 * RMSE_x = 2.4477 * RMSE_y$$
$$= 2.4477*RMSE/1.4142$$
$$Accuracy_r = 1.7308 * RMSE_r.$$

We can alternatively use the approximation of circular standard error at 95% confidence when RMSEx ≠ RMSEy and RMSEmin/RMSEmax is between 0.6 and 1.0 (FGDC-STD-007.3-1998 Appendix 3-a):

$$Accuracy_r \sim 2.4477 * 0.5 * (RMSE_x + RMSE_y)$$

Cadastral Map of 1832

Our choice of using the cadastral map of 1832 as the independent highly accurate dataset leads us to expect an initial effect in the calculation of NSSDA horizontal accuracy for early maps since this map is the result of combining two maps in different scales (local and regional). The first contains the data inside the water boundary around the old city, and the second , the area outside the same boundary (Figure 1).

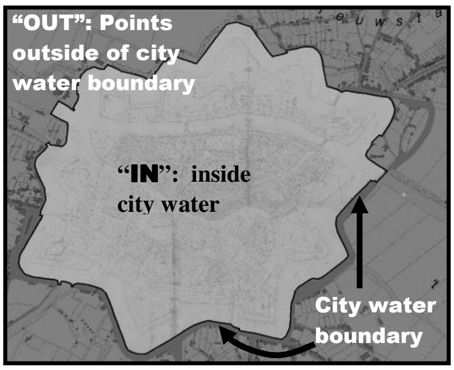

Figure 1. Areas inside (in light grey) and outside (in dark grey) of the city water boundary. The line defining this boundary marks inside/outside.

The horizontal NSSDA accuracy in the 1832 map tested with respect to the modern (2004) map was calculated at 3.73m at 95% confidence for the map as a whole, but values are clearly different when NSSDA is tested for areas inside and outside the city separately. NSSDA (at 95% confidence) is 1.94m inside the city as opposed to 6.88m outside. [10]

The sample: Selection and classification of test points

Twenty or more test points are required to conduct a statistically significant accuracy evaluation regardless of the size of the data set or area of coverage because twenty points make a computation at the 95% confidence level reasonable (Positional Accuracy Handbook, 3: 1999). However, due to the difficulty in finding common features between old and modern sources it was not always possible to obtain twenty points in every analysed category and class (see Table 1).

Once points were selected, they were grouped by categories according to whether they were inside the city water boundary (IN) or outside (OUT). This distinction was made to explore whether we find the same significant difference between these groups that we observed in the cadastral map 1832. We consider as well the possibility that the objects inside vs. outside might belong to different classes. These classes were defined according to the object they represent as indicated in Table 1 and Figure 2. The first eight classes belong to the category 'IN'. The last two classes belong to the category 'OUT'.

Table 1. Classes and number of tested points per class and per map.

Class	Description of location of points	Cat	Blaeu	Dh68	Dh68a	Dhslide	G35	Priorato	Total
Bastions	Bastions along the fortified city	**In**	33	32	32	32	33	29	**191**
Buildings	buildings (churches, mills, houses),	**In**	32	17	18	14	6	21	**108**
Bridges	Bridges	**In**	18	23	23	21	22	20	**127**
Streets	Streets intersections	**In**	179						**179**
Walled area	Buildings along the walled area	**In**	24	20	21	23	21	21	**130**
Water1	Water channels (internal)	**In**	46	45	44	45	44	40	**264**
Water2	Water channels (surrounding) internal	**In**	12	12	12	12	9	12	**69**
Water3	Water channels (surrounding) external	**In**	40	35	35	37	35	32	**214**
Parcels	Parcels intersections (external)	**Out**	16	41	42	42	20	16	**177**
Roads	Roads intersections (external)	**Out**	13	18	19	18	14	16	**98**
Total		**In_out**	**413**	**243**	**246**	**244**	**204**	**207**	**1557**

[10] We are aware that the difference in accuracy detected in the cadastral map 1832 (inside vs. outside) affects the results of accuracy for the early maps tested. However, we expect to find significant differences in depictions inside vs. outside the city boundaries in spite of the fact that the errors in the 1832 cadastral map limit the sensitivity of our probes.

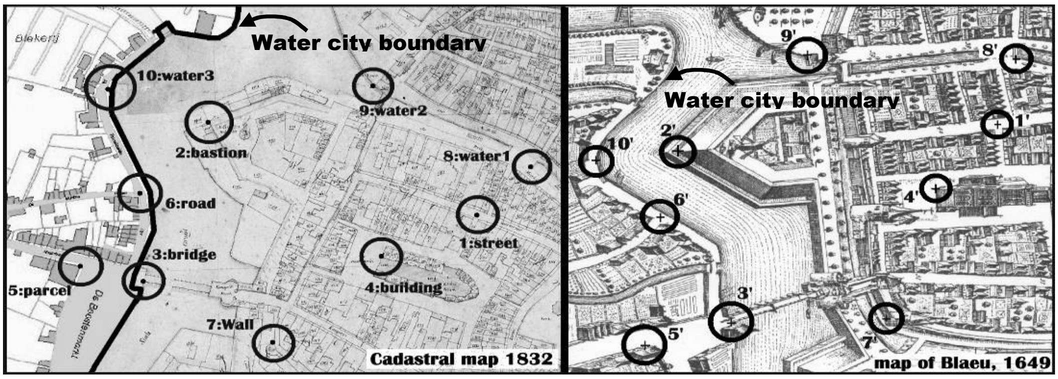

Figure 2. Example of features selected for comparison and analysis divided by classes according to the list shown in Table 1. The window on the left shows 10 points in the cadastral map 1832. The window on the right shows the corresponding points in the map of Blaeu (1649).

NSSDA horizontal accuracy per classes inside and outside city

Our assumption that the depiction inside the city differs from the depiction outside is confirmed when these areas are analyzed by category. For every class and map we have calculated the NSSDA values as shown in Table 2. As observed previously, classes inside the city all show lower values than classes outside (parcels and roads).

Regarding the results from the different transformations applied (see above) we note:

1. Different results are obtained from different transformations.
2. No matter which transformation is used, the maps Dh68, Dh68a and Dhslide show little difference in overall results (for all classes).
3. The application of Helmert and affine transformations results in very different accuracy estimations for the map of Blaeu in the classes water2, parcels and roads.
4. The biggest differences between the transformations are observed in the maps G35 and Priorato, which are also the most inaccurate maps.
5. The map G35 has very different accuracy estimations under the different transformations (differing by 2,8m to 24.2m), with exception of the class buildings (differing by 0,4m to 0,5m).
6. The map of Priorato has the highest error and also shows the clearest difference between the three transformations in all classes. Values for the classes bastions, walls,

water3, parcels and roads show the same tendencies observed in map G35. Conversely the map of Blaeu shows the smallest errors of the group.

Table 2. RMSE and NSSDA Horizontal Accuracy tested at a 95 % confidence level in meters per category (class) and per map for six maps of Zwolle, calculated for three different transformations: Two points (4p), Affine (6p) and Helmert (4p).

NSSDA horizontal accuracy in meters per class, in three different transformations										Total
Map of Blaeu 1649	**bastion**	**bridge**	**building**	**parcel**	**road**	**wall**	**water1**	**water2**	**water3**	**Blaeu**
NSSDA Two points	23.37	20.83	24.22	54.73	37.35	17.93	15.51	30.42	21.63	23.66
NSSDA Affine	20.56	15.41	26.39	80.55	50.89	14.28	9.27	51.27	20.56	25.69
NSSDA Helmert	21.81	18.37	27.84	79.90	52.14	16.32	10.67	52.15	21.40	26.28
Map Dh68 (1739-50)	**bastion**	**bridge**	**building**	**parcel**	**road**	**wall**	**water1**	**water2**	**water3**	**DH68**
NSSDA Two points	27.14	18.21	28.06	73.36	67.52	19.64	40.30	30.46	20.81	43.50
NSSDA Affine	27.45	18.46	29.48	74.81	63.14	16.00	34.81	33.35	18.72	42.60
NSSDA Helmert	26.72	19.04	28.89	71.76	62.69	15.73	34.15	32.72	18.50	41.50
Map Dh68a (1739)	**bastion**	**bridge**	**building**	**parcel**	**road**	**wall**	**water1**	**water2**	**water3**	**DH68a**
NSSDA Two points	29.02	17.57	27.16	76.63	66.84	20.69	40.95	31.18	21.71	44.70
NSSDA Affine	27.73	17.27	25.74	78.60	65.18	15.72	36.04	30.13	18.58	43.71
NSSDA Helmert	28.79	16.88	25.93	75.42	62.74	16.70	35.08	31.64	19.17	42.53
Map Dhslide (1739)	**bastion**	**bridge**	**building**	**parcel**	**road**	**wall**	**water1**	**water2**	**water3**	**Dhslide**
NSSDA Two points	23.20	23.35	30.33	58.43	60.29	21.45	31.47	36.34	23.62	37.66
NSSDA Affine	17.80	20.58	34.21	67.47	56.17	14.48	26.09	34.17	20.68	37.99
NSSDA Helmert	18.24	20.85	33.21	61.57	59.00	15.92	25.99	30.95	20.63	36.42
Map G35	**bastion**	**bridge**	**building**	**parcel**	**road**	**wall**	**water1**	**water2**	**water3**	**G35**
NSSDA Two points	41.72	41.13	34.30	83.63	74.65	28.91	42.56	63.99	35.78	50.41
NSSDA Affine	31.55	29.88	33.86	63.92	59.22	21.37	30.15	32.72	24.06	37.05
NSSDA Helmert	37.44	38.46	33.12	75.15	72.28	25.92	34.94	43.68	30.88	44.40
Priorato 1673	**bastion**	**bridge**	**building**	**parcel**	**road**	**wall**	**water1**	**water2**	**water3**	**Priorato**
NSSDA Two points	66.71	68.50	89.65	102.28	94.51	63.23	65.04	93.27	71.73	77.75
NSSDA Affine	44.64	43.28	78.41	76.87	77.22	48.16	64.04	68.29	42.35	60.55
NSSDA Helmert	56.06	47.04	73.52	93.81	87.98	54.29	60.66	71.19	58.24	66.58

NSSDA horizontal accuracy was calculated as: ~ $2.4477 * 0.5 * (RMSE_x + RMSE_y)$ since $RMSE_x \neq RMSE_y$. NSSDA statistic for the class streets in the map pf Blaeu (1649) reports 20m,13.7 asnd 14.3 m for Two points, Affine and Helmert transformations respectively.

Priority in the depiction of features in old maps

In this analysis we assume that positional error is inverse to the importance mapmakers attached to the depiction of objects in old maps, which means that a high error indicates low priority and low error high priority. This then will be measured by the NSSDA horizontal accuracy standard. This assumption could be also affected by the fact that it could be difficult to measure the position of some objects (especially outside the city) which are difficult to interpret and link to the cadastral or modern map resulting in fewer identifiable points in this area of the maps.

We therefore conclude that there is indeed a priority in the depiction of features from map to map and within the same map. We observe smaller error in the depiction of features inside the city than outside the city. This can be explained in relation to the general purpose of the map, which is to show the fortified city rather than its surroundings, and which is then reflected by the accuracy with which the mapmaker depicts features inside and outside the city boundary. This explains the fact that parcels and (external) roads are reported as the most inaccurate features per map.

Conversely, the area inside the city ('IN') the classes of walls, bridges and 'water3' are reported as the most accurate features in each map with some few exceptions. On the other hand, the external boundaries of canals surrounding the city ('water2') are the most inaccurate classes in all maps when compared to other classes.

Together with the lesser accuracy reported external parcels and roads in ('OUT') in all maps, we note that there is no significant difference between the two kinds of features.

The Figure 3 (table with scale in meters) shows the priority in the depiction we found for the set of six maps of Zwolle. Using this scale we immediately notice that the map of Priorato shows the largest errors (located at the end of the scale) but also the smallest range of error. Conversely, the maps of Blaeu and Dhslide show the smallest error and the least variation in error in almost all classes when compared to other maps of the set.

Within the same set, the maps Dh68 and DH68a show very similar accuracy (see Table 2) per map and per class. This is expected since the two maps are part of the same process of mapmaking, in which the map DH68a was an earlier phase of the map DH68. The difference shows a slight improvement in the accuracy in the later phase (Dh68). However, the improvement turns out to be modest when one looks at the values of classes separately (Table 2). Some similarity between Dh68 (1739-1750) and DH68a (1739) on the one hand and DHslide (1739) on the other is also observed. This can be explained by the fact that the maps were made at about the same time.

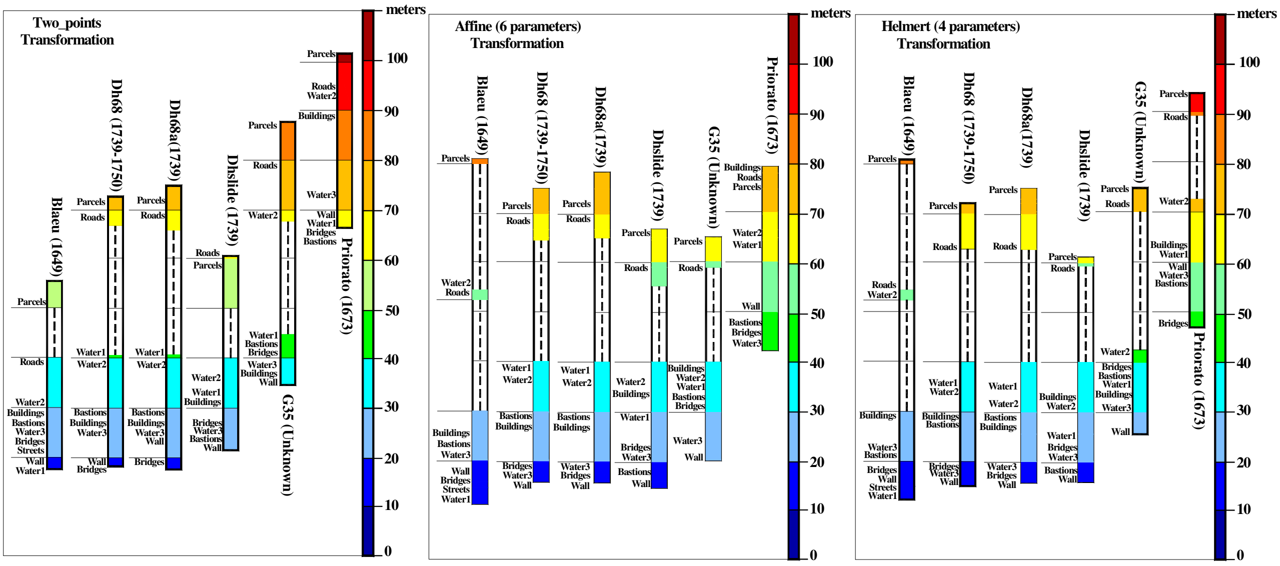

Figure 3. The table (above) shows the priority in the depiction of classes per map varying from highest priority (in the lowest part of the scales) to lowest priority (features in the upper part of the scale). The colors of the scale show the variation of error according the respective scale in meters (right), varying from blue (low error) to red (high error). A discontinuity in the error values is shown by the dotted line. This indicates in most of the cases (except G35) a remarkable difference between classes inside the city and outside the city.

Conclusions

In spite of the limitations and constraints which accompany the use of early sources in contrast to situations where modern and highly accurate sources are used, we have calculated the NSSDA horizontal accuracy to help us to interpret and better understand the level of accuracy in early maps

There is a clear difference between features depicted inside the city boundary as opposed to the features outside the city boundary. We believe that these differences observed in early maps also reflect real differences in the priority of the depiction of the features in the maps. This priority is evident not only from map to map but also within the same map, where the features inside the city are depicted more accurately (therefore with higher priority) than those outside the city (lower priority). This can be explained on the basis of the general purpose and the function of the map. Probably none of these maps were made with the purpose to show features outside the city boundary accurately .

The sample size and the geo-referencing method used to determine accuracy has an effect on the results. We believe that the more homogeneous the sizes of the samples per category, the more representative and reliable the results will be. For that reason, we consider that the best way to measure accuracy of the area outside the city would probably be to use a larger and more uniform dataset of points covering the minimum requirement of 20 points per class.[11] This emphasizes the need to distinguish the categories and subcategories (classes) inside and outside the city walls. The more detailed the analysis, the more test points we require.

As future work, these results must be confronted with the historical evidence about the mapmaking process. We intend to seek an explanation for this in the land survey and mapmaking techniques used by the mapmakers.

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[11] In fact we were not able to find this required minimum in all maps and this may have affected the results.