Conditional entropy as a measure of intelligibility

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Outline

- The languages in concern are the three mainland Scandinavian languages: Swedish, Danish, Norwegian
- Can be considered dialects which are separate languages only for political and historical reasons
- Enables semi-communication: Inter-Scandinavian communication in each speaker's own language
- Tendency that mutual intelligibility has deteriorated
- How can you measure this deterioration, i.e. the linguistic distance between the languages?

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- Measure phonetic distances among Scandinavian languages
- Discover regularities in phonetic correspondences
- Knowledge of correspondences enables word prediction
- How can you measure the difficulty of these predictions?
- By using conditional entropy (CE)!

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- ..probabilities of sound correspondences can be measured
- The minimum number of bits needed to encode one language based on the other (the conditional entropy) can then be calculated
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Statistical entropy, overview

- Entropy measures the amount of information in a random variable..
- ..i.e. the degree of freedom in a given situation
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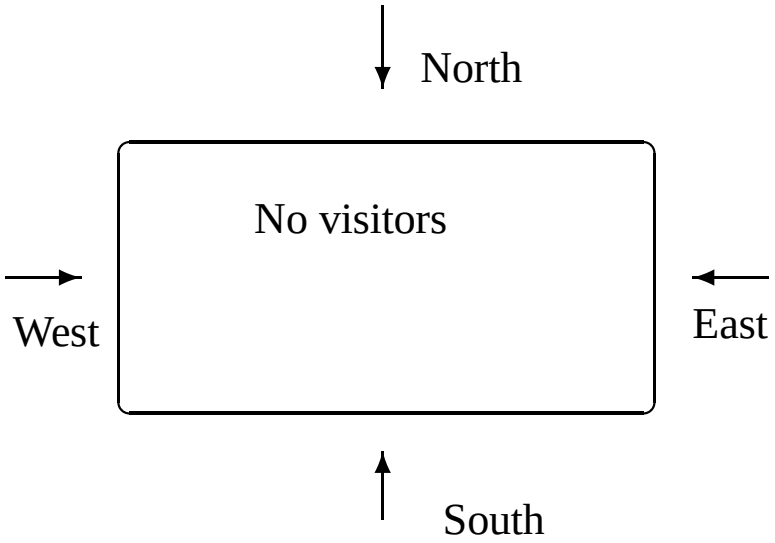
The lookout example

If a visitor is approaching, the lookout needs to report the direction the visitor is coming from, i.e. one of five messages:



These options can be thought of as bits: 000, 001, 010, 011 and 100. All codes have three bits.

Example: code length

Assuming that every message is equally likely, the binary log gives us the code length:

$$\text{code length} = \lceil \log_2 |M| \rceil, \text{ where } M \text{ are the messages}$$

What happens when you add frequency?

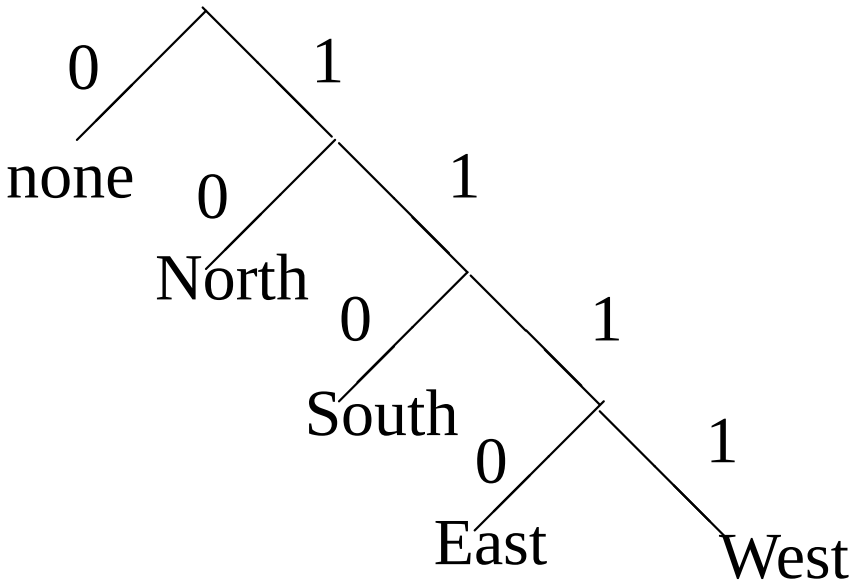
message	rel. freq.
no visitor	99%
North	0.5%
South	0.25%
East, West	0.125%

Example: expected Code Length

We now calculate the code length:

message	code length	rel. freq.	expected bit length
no visitor	1	0.99	0.99
North	2	0.005	0.01
South	3	0.0025	0.0075
East	4	0.00125	0.005
West	4	0.00125	0.005
Total			1.0175

Example: a code tree



message	code
no visitor	0
North	10
South	110
East	1110
West	1111

Entropy

The optimal code cannot be compressed further than the **entropy** (informational uncertainty) of the dataset:

$$H(S) = -\sum_{i \in S} p_i \log_2 p_i$$

message	p_i	$-\log p_i$	$p_i \log p_i$
no visitor	0.99	0.004	0.0044
North	0.005	2.3	0.0115
South	0.0025	2.6	0.0065
East	0.00125	2.9	0.0036
West	0.00125	2.9	0.0036
Total			0.021

- Think of entropy as the ”20 questions” game! You need to ask 0.021 yes/no questions on average to identify the message

Entropy reduction

- By adding knowledge to the system, one reduces the uncertainty. The information gain can be quantified by comparing the total entropies of the original system and the final system.
- Suppose visitors never come on Mondays. Then adding information about the day of the week will reduce the entropy:

Day	P	Entropy
Mondays	0.143	0
Other	0.857	0.021
Total		0.018

Outline

- The joint entropy of a pair of random variables is the amount of info needed on average to specify both their values:

$$H(X,Y) = -\sum_{x \in X} \sum_{y \in Y} p(x,y) log p(x,y)$$

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Overview and explanation on conditional entropy

- CE is always calculated in relation to other information
- CE relies on conditional probabilities
- CE of Y given X is the joint entropy of X and Y minus the entropy of X:

$$H(Y \mid X) = H(X, Y) - H(X)$$

- As opposed to joint entropy, CE is not symmetrical:

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Measuring CE

- $H(X \mid Y)$ is the uncertainty in X given knowledge of Y.
- CE measures how much entropy a random variable Y has remaining if the value of a second random variable X is known
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Example: Danish realizations of a Swedish phoneme

Table: Conditional probabilities for Danish phonemes given Swedish /a/

Danish →	ə	a	ɒ	Others
Swedish ↓				
a	0.45	0.14	0.10	0.31
o				
u				
..etc				

Using probabilities to calculate CE

- Entropy H $(P(D \mid a))$

$$H = -\sum_{d \in D, a} p(d, a) log_2(d \mid a)$$

$$H = -(0.45 * log_2 0.45) + (0.14 * log_2 0.14) + (0.10 * log_2 0.10) + (0.31 * log_2 0.31)$$

- H(D|a) = 1.775 bits of information
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- The sample corpus: 206 Danish-Swedish word pairs, some non-cognates
- Due to insertions and deletions, the word length is sometimes different. I use a filler symbol to account for this.
- S means source(Swedish) and T target (Danish)
- H(S|T) = 2.29
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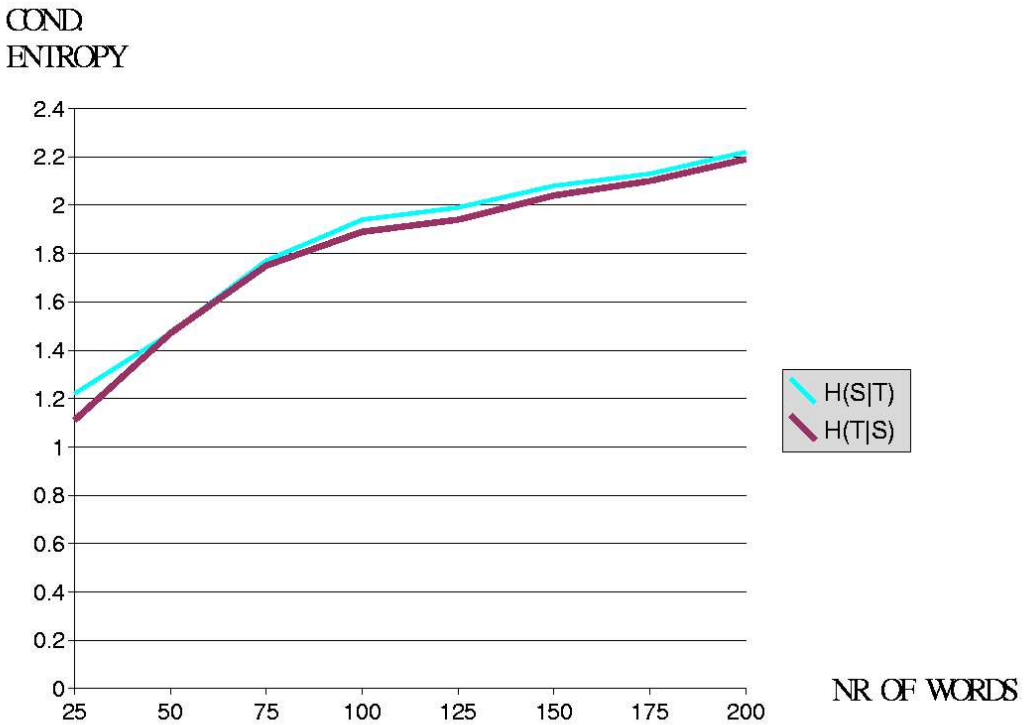
Uniform distribution of CE



- For 25 words: H(S|T) = 1.22, H(T|S) = 1.11
- 200 words: H(S|T)= 2.22, H(T|S)= 2.19

What is then a suffi cient data size?

- Hard to tell..
- But every phoneme should occur at least 10 times(roughly)
- By then, the probabilities should have stabilized and be reliable measures for the languages as a whole

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Future work

- Complete the corpus: a core corpus with 3 parts: function words, formal content words and informal content words
- Add information about date of introduction into the language
- For the entropy experiments: extend the corpus by adding data from a Swedish speech corpus that is phonetically transcribed
- Look at the correspondence between bigrams
- Analyze the results: how does the style of language affect entropy?
- Look at date of introduction: how does this affect the difference between the languages in terms of entropy?

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