The intersection of Finite State Automata and Definite Clause Grammars

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Abstract

Bernard Lang defines parsing as the calculation of the intersection of a FSA (the input) and a CFG. Viewing the input for parsing as a FSA rather than as a string combines well with some approaches in speech understanding systems, in which parsing takes a word lattice as input (rather than a word string). Furthermore, certain techniques for robust parsing can be modelled as finite state transducers.

In this paper we investigate how we can generalize this approach for unification grammars. In particular we will concentrate on how we might the calculation of the intersection of a FSA and a DCG. It is shown that existing parsing algorithms can be easily extended for FSA inputs. However, we also show that the termination properties change drastically: we show that it is undecidable whether the intersection of a FSA and a DCG is empty (even if the DCG is off-line parsable).

Furthermore we discuss approaches to cope with the problem.

1 Introduction

In this paper we are concerned with the syntactic analysis phase of a natural language understanding system. Ordinarily, the input of such a system is a sequence of words. However, following Bernard Lang we argue that it might be fruitful to take the input more generally as a *finite state automaton (FSA)* to model cases in which we are uncertain about the actual input. Parsing uncertain input might be necessary in case of ill-formed textual input, or in case of speech input.

For example, if a natural language understanding system is interfaced with a speech recognition component, chances are that this compenent is uncertain about the actual string of words that has been uttered, and thus produces a *word lattice* of the most promising hypotheses, rather than a single sequence of words. FSA of course generalizes such word lattices.

As another example, certain techniques to deal with ill-formed input can be characterized as finite state transducers (Lang, 1989); the composition of an input string with such a finite state transducer results in a FSA that can then be input for syntactic parsing. Such an approach allows for the treatment of missing, extraneous, interchanged or misused words (Teitelbaum, 1973; Saito and Tomita, 1988; Nederhof and Bertsch, 1994).

Such techniques might be of use both in the case of written and spoken language input. In the latter case another possible application concerns the treatment of phenomena such as repairs (Carter, 1994).

Note that we allow the input to be a full FSA (possibly including cycles, etc.) since some of the above-mentioned techniques indeed result in cycles. Whereas an ordinary word-graph always defines a finite language, a FSA of course can easily define an infinite number of sentences. Cycles might emerge to treat unknown sequences of words, i.e. sentences with unknown parts of unknown lengths (Lang, 1988).

As suggested by an ACL reviewer, one could also try to model haplology phenomena (such as the 's in English sentences like 'The chef at Joe's hat', where 'Joe's' is the name of a restaurant) using a finite state transducer. In a straightforward approach this would also lead to a finite-state automaton with cycles.

It can be shown that the computation of the intersection of a FSA and a CFG requires only a minimal generalization of existing parsing algorithms. We simply replace the usual string positions with the names of the states in the FSA. It is also straightforward to show that the complexity of this process is cubic in the number of states of the FSA (in the case of ordinary parsing the number of states equals $n+1$) (Lang, 1974; Billot and Lang, 1989) (assuming the right-hand-sides of grammar rules have at most two categories).

In this paper we investigate whether the same techniques can be applied in case the grammar is a constraint-based grammar rather than a CFG. For specificity we will take the grammar to be a *Definite Clause Grammar* (DCG) (Pereira and Warren, 1980). A DCG is a simple example of a family of constraint-based grammar formalisms that are widely used in natural language analysis (and generation). The main findings of this paper can be extended to other members of that family of constraint-based grammar formalisms.

2 The intersection of a CFG and a FSA

The calculation of the intersection of a CFG and a FSA is very simple (Bar-Hillel et al., 1961). The (context-free) grammar defining this intersection is simply constructed by keeping track of the state names in the non-terminal category symbols. For each rule $X_0 \rightarrow X_1 \dots X_n$ there are rules $\langle X_0 q_0 q\rangle \rightarrow \langle X_1 q_0 q_1\rangle\langle X_2 q_1 q_2\rangle \dots \langle X_n q_{n-1} q\rangle$, for all $q_0 \dots q_n$. Furthermore for each transition $\delta(q_i, \sigma) = q_k$ we have a rule $\langle \sigma q_i q_k\rangle \rightarrow \sigma$. Thus the intersection of a FSA and a CFG is a CFG that exactly derives all parse-trees. Such a grammar might be called the parse-forest grammar.

Although this construction shows that the intersection of a FSA and a CFG is itself a CFG, it is not of practical interest. The reason is that this construction typically yields an enormous amount of rules that are 'useless'. In fact the (possibly enormously large) parse forest grammar might define an empty language (if the intersection was empty). Luckily 'ordinary' recognizers/parsers for CFG can be easily generalized to construct this intersection yielding (in typical cases) a much smaller grammar. Checking whether the intersection is empty or not is then usually very simple as well: only in the latter case will the parser terminate succesfully.

To illustrate how a parser can be generalized to accept a FSA as input we present a simple top-down parser.

A context-free grammar is represented as a definite-clause specification as follows. We do not wish to define the sets of terminal and non-terminal symbols explicitly, these can be understood from the rules that are defined using the relation `rule/2`, and where symbols of the rhs are prefixed with '-' in the case of terminals and '+' in the case of non-terminals. The relation `top/1` defines the start symbol. The language $L' = a^n b^n$ is defined as:

```
top(s).

rule(s,[-a,+s,-b]).  rule(s,[]).
```

In order to illustrate how ordinary parsers can be used to compute the intersection of a FSA and a CFG consider first the definite-clause specification of a top-down parser. This parser runs in polynomial time if implemented using Earley deduction or XOLDT resolution (Warren, 1992). It is assumed that the input string is represented by the `trans/3` predicate.

```
parse(P0,P) :-
    top(Cat), parse(+Cat,P0,P).

parse(-Cat,P0,P) :-
    trans(P0,Cat,P),
    side_effect(p(Cat,P0,P) --> Cat).
parse(+Cat,P0,P) :-
    rule(Cat,Ds),
    parse_ds(Ds,P0,P,His),
    side_effect(p(Cat,P0,P) --> His).

parse_ds([],P,P,[]).
parse_ds([H|T],P0,P,[p(H,P0,P1)|His]) :-
    parse(H,P0,P1),
    parse_ds(T,P1,P,His).
```

The predicate `side_effect` is used to construct the parse forest grammar. The predicate always succeeds, and as a side-effect asserts that its argument is a rule of the parse forest grammar. For the sentence 'a a b b' we obtain the parse forest grammar:

```
p(s,2,2) --> [].
p(s,1,3) -->
    [p(-a,1,2),p(+s,2,2),p(-b,2,3)].
p(s,0,4) -->
    [p(-a,0,1),p(+s,1,3),p(-b,3,4)].
p(a,1,2) --> a.
p(a,0,1) --> a.
p(b,2,3) --> b.
p(b,3,4) --> b.
```

The reader easily verifies that indeed this grammar generates (a isomorphism of) the single parse tree of this example, assuming of course that the start symbol for this parse-forest grammar is `p(s,0,4)`. In the parse-forest grammar, complex symbols are non-terminals, atomic symbols are terminals.

Next consider the definite clause specification of a FSA. We define the transition relation using the relation `trans/3`. For start states, the relation

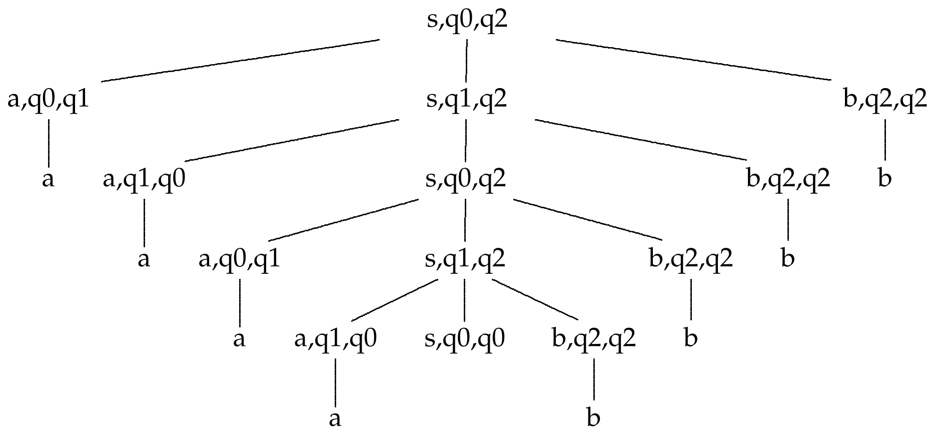
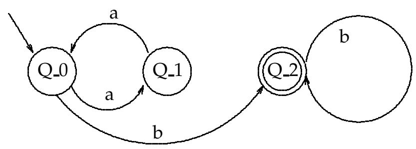


Figure 1: A parse-tree extracted from the parse forest grammar

start/1 should hold, and for final states the relation final/1 should hold. Thus the following FSA, defining the regular language $L = (aa)^* b^+$ (i.e. an even number of a's followed by at least one b) is given as:




```
start(q0).  final(q2).

trans(q0,a,q1).  trans(q1,a,q0).
trans(q0,b,q2).  trans(q2,b,q2).
```

Interestingly, nothing needs to be changed to use the same parser for the computation of the intersection of a FSA and a CFG. If our input 'sentence' now is the definition of `trans/3` as given above, we obtain the following parse forest grammar (where the start symbol is `p(s,q0,q2)`):

```
p(s,q0,q0) --> [].
p(s,q1,q1) --> [].
p(s,q1,q2) -->
    [p(-a,q1,q0),p(+s,q0,q0),p(-b,q0,q2)].
p(s,q0,q2) -->
    [p(-a,q0,q1),p(+s,q1,q2),p(-b,q2,q2)].
p(s,q1,q2) -->
    [p(-a,q1,q0),p(+s,q0,q2),p(-b,q2,q2)].
p(a,q0,q1) --> a.
p(a,q1,q0) --> a.
p(b,q0,q2) --> b.
p(b,q2,q2) --> b.
```

Thus, even though we now use the same parser for an infinite set of input sentences (represented by the FSA) the parser still is able to come up with a parse forest grammar. A possible derivation for this grammar constructs the following (abbreviated) parse tree in figure 1. Note that the construction of Bar Hillel would have yielded a grammar with 88 rules.

3 The intersection of a DCG and a FSA

In this section we want to generalize the ideas described above for CFG to DCG.

First note that the problem of calculating the intersection of a DCG and a FSA can be solved trivially by a generalization of the construction by (Bar-Hillel et al., 1961). However, if we use that method we will end up (typically) with an enormously large forest grammar that is not even guaranteed to contain solutions [1]. Therefore, we are interested in methods that only generate a small subset of this; e.g. if the intersection is empty we want an empty parse-forest grammar.

The straightforward approach is to generalize existing recognition algorithms. The same techniques that are used for calculating the intersection of a FSA and a CFG can be applied in the case of DCGs. In order to compute the intersection of a DCG and a FSA we assume that FSA are represented as before. DCGs are represented using the same notation we used for context-free grammars, but now of course the category symbols can be first-order terms of arbitrary complexity (note that without loss of generality we don't take into account DCGs having exter-

[1] In fact, the standard compilation of DCG into Prolog clauses does something similar using variables instead of actual state names. This also illustrates that this method is not very useful yet; all the work has still to be done.

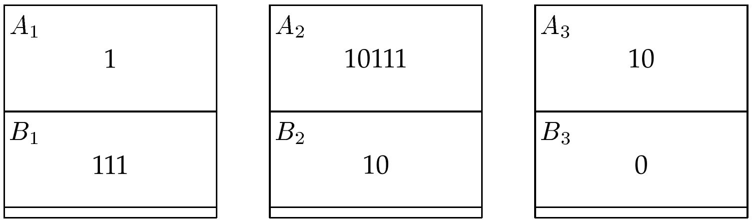


Figure 2: Instance of a PCP problem.

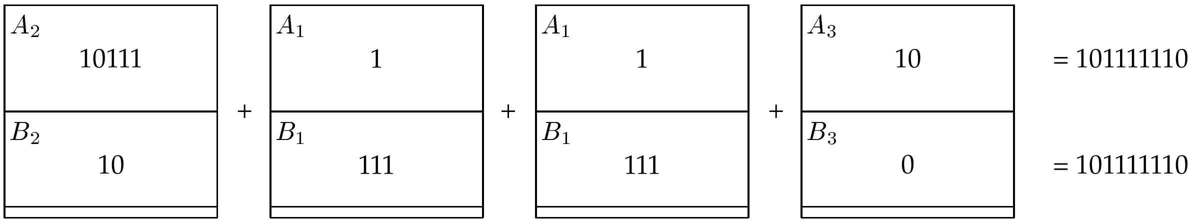


Figure 3: Illustration of a solution for the PCP problem of figure 2.

nal actions defined in curly braces).

But if we use existing techniques for parsing DCGs, then we are also confronted with an undecidability problem: the recognition problem for DCGs is undecidable (Pereira and Warren, 1983). A fortiori the problem of deciding whether the intersection of a FSA and a DCG is empty or not is undecidable.

This undecidability result is usually circumvented by considering subsets of DCGs which can be recognized effectively. For example, we can restrict the attention to DCGs of which the context-free skeleton does not contain cycles. Recognition for such 'off-line parsable' grammars is decidable (Pereira and Warren, 1983).

Most existing constraint-based parsing algorithms will terminate for grammars that exhibit the property that for each string there is only a finite number of possible derivations. Note that off-line parsability is one possible way of ensuring that this is the case.

This observation is not very helpful in establishing insights concerning interesting subclasses of DCGs for which termination can be guaranteed (in the case of FSA input). The reason is that there are now two sources of recursion: in the DCG and in the FSA (cycles). As we saw earlier: even for CFG it holds that there can be an infinite number of analyses for a given FSA (but in the CFG this of course does not imply undecidability).

3.1 Intersection of FSA and off-line parsable DCG is undecidable

I now show that the question whether the intersection of a FSA and an off-line parsable DCG is empty is undecidable. A yes-no problem is *undecidable* (cf. (Hopcroft and Ullman, 1979, pp.178-179)) if there is no algorithm that takes as its input an *instance* of the problem and determines whether the answer to that instance is 'yes' or 'no'. An instance of a problem consists of a particular choice of the *parameters* of that problem.

I use Post's Correspondence Problem (PCP) as a well-known undecidable problem. I show that if the above mentioned intersection problem were decidable, then we could solve the PCP too. The following definition and example of a PCP are taken from (Hopcroft and Ullman, 1979)[chapter 8.5].

An instance of PCP consists of two lists, $A = v_1 \dots v_k$ and $B = w_1 \dots w_k$ of strings over some alphabet Σ. This instance has *a solution* if there is any sequence of integers $i_1 \dots i_m$, with $m \geq 1$, such that

$$v_{i_1}, v_{i_2}, \dots, v_{i_m} = w_{i_1}, w_{i_2}, \dots, w_{i_m}.$$

The sequence $i_1, \dots, i_m$ is a solution to this instance of PCP. As an example, assume that $\Sigma = \{0, 1\}$. Furthermore, let $A = \langle 1, 10111, 10 \rangle$ and $B = \langle 111, 10, 0 \rangle$. A solution to this instance of PCP is the sequence 2,1,1,3 (obtaining the sequence 101111110). For an illustration, cf. figure 3.

Clearly there are PCP's that do not have a solution. Assume again that $\Sigma = \{0, 1\}$. Furthermore let $A = \langle 1 \rangle$ and $B = \langle 0 \rangle$. Clearly this PCP does not have a solution. In general, however, the problem

```
trans(q0,x,q0).   start(q0).   final(q0).   % FSA

top(s).                                     % start symbol DCG

rule(s,[-r(X,[],X,[])]).                    % require A's and B's match

rule(r(A0,A,B0,B),[-r(A0,A1,B0,B1),         % combine two sequences of
                   -r(A1,A,B1,B)]).         % blocks

rule(r([1|A],        A,[1,1,1|B],B),[+x]).  % block A1/B1
rule(r([1,0,1,1,1|A],A,[1,0|B],  B),[+x]).  % block A2/B2
rule(r([1,0|A],      A,[0|B],    B),[+x]).  % block A3/B3
```

Figure 4: The encoding for the PCP problem of figure 2.

whether some PCP has a solution or not is not decidable. This result is proved by (Hopcroft and Ullman, 1979) by showing that the halting problem for Turing Machines can be encoded as an instance of Post's Correspondence Problem.

First I give a simple algorithm to encode any instance of a PCP as a pair, consisting of a FSA and an off-line parsable DCG, in such a way that the question whether there is a solution to this PCP is equivalent to the question whether the intersection of this FSA and DCG is empty.

Encoding of PCP.

1. For each $1 \leq i \leq k$ (k the length of lists A and B) define a DCG rule (the $i-th$ member of A is $a_1 \ldots a_m$, and the $i-$th member of B is $b_1 \ldots b_n$): $r([a_1 \ldots a_m|A], A, [b_1 \ldots b_n|B], B) \rightarrow [x]$.
2. Furthermore, there is a rule $r(A_0, A, B_0, B) \rightarrow r(A_0, A_1, B_0, B_1), r(A_1, A, B_1, B)$.
3. Furthermore, there is a rule $s \rightarrow r(X, [\,], X, [\,])$. Also, s is the start category of the DCG.
4. Finally, the FSA consists of a single state q which is both the start state and the final state, and a single transition $\delta(q, x) = q$. This FSA generates x^*.

Observe that the DCG is off-line parsable.

The underlying idea of the algorithm is really very simple. For each pair of strings from the lists A and B there will be one lexical entry (deriving the terminal x) where these strings are represented by a difference-list encoding. Furthermore there is a general combination rule that simply concatenates A-strings and concatenates B-strings. Finally the rule for s states that in order to construct a succesful top category the A and B lists must match.

The resulting DCG, FSA pair for the example PCP is given in figure 4:

Proposition The question whether the intersection of a FSA and an off-line parsable DCG is empty is undecidable.

Proof. Suppose the problem *was* decidable. In that case there would exist an algorithm for solving the problem. This algorithm could then be used to solve the PCP, because a PCP π has a solution if and only if its encoding given above as a FSA and an off-line parsable DCG is not empty. The PCP problem however is known to be undecidable. Hence the intersection question is undecidable too.

3.2 What to do?

The following approaches towards the undecidability problem can be taken:

- limit the power of the FSA
- limit the power of the DCG
- compromise completeness
- compromise soundness

These approaches are discussed now in turn.

Limit the FSA Rather than assuming the input for parsing is a FSA in its full generality, we might assume that the input is an ordinary word graph (a FSA without cycles).

Thus the techniques for robust processing that give rise to such cycles cannot be used. One example is the processing of an unknown sequence of words, e.g. in case there is noise in the input and it is not clear how many words have been uttered during this noise. It is not clear to me right now what we loose (in practical terms) if we give up such cycles.

Note that it is easy to verify that the question whether the intersection of a word-graph and an off-line parsable DCG is empty or not is decidable since it reduces to checking whether the DCG derives one of a finite number of strings.

Limit the DCG Another approach is to limit the size of the categories that are being employed. This is the GPSG and F-TAG approach. In that case we are not longer dealing with DCGs but rather with CFGs (which have been shown to be insufficient in general for the description of natural languages).

Compromise completeness Completeness in this context means: the parse forest grammar contains all possible parses. It is possible to compromise here, in such a way that the parser is guaranteed to terminate, but sometimes misses a few parse-trees.

For example, if we assume that each edge in the FSA is associated with a probability it is possible to define a threshold such that each partial result that is derived has a probability higher than the threshold. Thus, it is still possible to have cycles in the FSA, but anytime the cycle is 'used' the probability decreases and if too many cycles are encountered the threshold will cut off that derivation.

Of course this implies that sometimes the intersection is considered empty by this procedure whereas in fact the intersection is not. For any threshold it is the case that the intersection problem of off-line parsable DCGs and FSA is decidable.

Compromise soundness Soundness in this context should be understood as the property that all parse trees in the parse forest grammar are valid parse trees. A possible way to ensure termination is to remove all constraints from the DCG and parse according to this context-free skeleton. The resulting parse-forest grammar will be too general most of the times.

A practical variation can be conceived as follows. From the DCG we take its context-free skeleton. This skeleton is obtained by removing the constraints from each of the grammar rules. Then we compute the intersection of the skeleton with the input FSA. This results in a parse forest grammar. Finally, we add the corresponding constraints from the DCG to the grammar rules of the parse forest grammar.

This has the advantage that the result is still sound and complete, although the size of the parse forest grammar is not optimal (as a consequence it is not guaranteed that the parse forest grammar contains a parse tree). Of course it is possible to experiment with different ways of taking the context-free skeleton (including as much information as possible / useful).

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